1964

A mathematical and experimental analysis of a catenoidal horn used as a coupling device for underwater acoustic transducers.

Dykeman, Charles J.

Monterey, California: U.S. Naval Postgraduate School

A MATHEMATICAL AND EXPERIMENTAL ANALYSIS OF A CATENOIDAL HORN USED AS A COUPLING DEVICE FOR UNDERWATER ACOUSTIC TRANSDUCERS

CHARLES J. DYKEMAN

66

A MATHEMATICAL AND EXPERIMENTAL ANALYSIS

OF A CATENOIDAL HORN USED AS A COUPLING

DEVICE FOR UNDERWATER ACOUSTIC TRANSDUCERS

* * * * * * *

Charles J. Dykeman

A MATHEMATICAL AND EXPERIMENTAL ANALYSIS
OF A CATENOIDAL HORN USED AS A COUPLING
DEVICE FOR UNDERWATER ACOUSTIC TRANSDUCERS

by

Charles J. Dykeman

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1 9 6 4

A MATHEMATICAL AND EXPERIMENTAL ANALYSIS OF A CATENOIDAL HORN USED AS A COUPLING DEVICE FOR UNDERWATER ACOUSTIC TRANSDUCERS

by

Charles J. Dykeman

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING
from the
United States Naval Postgraduate School

ABSTRACT

With the great demand for extending the ranges of active sonar on today's ships, analysis into lower frequency transmission and better power transfer is of great importance. This paper uses wave theory to analyze a steel horn and its effect on impedance, frequency response, and directivity of an electro-mechanical transducer.

The horn was found to make the transducer a broader band device but only a small gain was observed in power output.

The author wishes to express his appreciation to the Transducer Department of the Submarine Signal Division, Raytheon Company for the free use of their facilities and the excellent cooperation given in the successful completion of this paper. He especially wishes to thank Mr. Joseph Kuzneski, Mr. Serge Visotsky and Mr. Claude Ledoux for their valuable assistance.

TABLE OF CONTENTS

Chapter	Title	Page
I	Introduction	1
II	The General Wave Equation	7
III	A. Application of Wave Equation to the Transducer without Horn	15
	B. Specific Acoustic Impedance for the Transducer without Horn	19
IV	A. Application of Wave Equation to the Transducer with Horn	24
	B. Specific Acoustic Impedance for the Transducer with Horn	32
V	Theoretical Conclusions	44
VI	A. Introduction	49
	B. Description of Transducer	49
	C. Description of Horn	49
	D. Admittance Experiment	49
	E. Vector Impedance Locus Plot Experiment	63
	F. Frequency Response	67
	G. Beam Patterns	75
VII	Conclusions	77
APPENDIX	I	78
BIBLIOGRAPHY		79
SELECTED READING		80

LIST OF ILLUSTRATIONS

Figure		Page
1-1	Major Elements of a Transducer	3
1-2	Transducer Equivalent Circuit	3
1-3	Attenuation of Sound Waves in Water	5
2-1	Volume Element in Coordinate System	8
3-1	Spherical Coordinate System	15
3-2	Specific Acoustic Impedance D = 2 in.	21
3-3	Specific Acoustic Impedance D = 6 in.	22
3-4	Specific Acoustic Impedance D = 10 in.	23
4-1	Volume Element of a Horn	25
4-2	Graphical Layout of Mathematical Horn	28
4-3	Surface of a Transducer Piston	36
4-4	Specific Acoustic Impedance Finite Horn D = 0 in.	40
4-5	Specific Acoustic Impedance Finite Horn D = 2 in.	41
4-6	Specific Acoustic Impedance Finite Horn D = 6 in.	42
4-7	Specific Acoustic Impedance Finite Horn D = 10 in.	43
5-1	Specific Acoustic Impedance Infinite Horn	45
5-2	Comparison of the Resistive Loading Effect	46
5-3	Specific Acoustic Impedance of Various Media	48
6-1	View of Horn Coupled to Transducer	50
6-2	View of Horn Coupled to Transducer	51
6-3	Dimensional View of the Horn	53
6-4	Fabrication Views of Horn	54
6-5	Fabrication Views of Horn	55
6-6	Block Diagram of the Admittance Bridge	57

List of Illustrations

Figure		Page
6-7	Top View of Test Tank with Transducer	58
6-8	Side View of Test Tank with Transducer	58
6-9	Impedance Data for No Horn in Water	59
6-10	Impedance Data for Horn in Water	60
6-11	Comparison of Resistive Effects	61
6-12	Block Diagram of the Impedance Bridge	64
6-13	VILP for No Horn in Water	65
6-14	VILP for Horn in Water	66
6-15	Block Diagram of Raytrac System	68
6-16	Experimental Frequency Response Curve for Transducer without Horn	70
6-17	Experimental Frequency Curve for Transducer with Horn	71
6-18	Dimentional View of Calibrated Hydrophone and Transducer in Test Tank	72
6-19	Frequency Response Curves	74
6-20	Directivity Index Curves	76

CHAPTER I

INTRODUCTION

The primary objective of a sonar system is to convert electrical energy into acoustic energy for the purpose of transmitting intelligence in water. The most important purpose being in the detection of underwater craft. To accomplish this, electro-mechanical transducers are used to convert electrical pulses into underwater sound waves which can be transmitted through the sea to a target and back to the source. The transducer is then used to reconvert the incoming sound energy to be monitered and used for detection. Many types of transducers have been used such as underwater bells, microphones, magnetostrictive, piezo crystal, and more currently piezo ceramic transducers.

In the early years of sonar, the ranges of transmission were extremely limited as were the associated weapon systems. Today the ranges needed have been greatly extended and have put a limitation on the effectiveness of advanced weapon systems. Methods of obtaining greater ranges of transmission are therefore under intensive study. There are many facets that can be investigated in this area. Greater ranges can be achieved by using larger systems, more sophisticated systems, systems drawing more power, or any combination of these. Also research on new principles or ideas could prove fruitful. One other area could be an investigation into increasing the effectiveness of existing systems. One branch of the later will be investigated in this paper.

Before discussing the objectives of this paper, it would be advantageous to briefly review a general type of electro-mechanical ceramic transducer.

Generally a ceramic transducer is made up of three major parts: the back mass, the ceramic element, and the front mass. The major elements of such a transducer can be seen in Fig. 1-1. The back mass is usually a large solid mass of steel with a high moment of inertia. The purpose of the high moment of inertia is to prevent radiation of energy to the rear of the transducer. Behind the back mass there is usually an absorbing material to take up any radiation to the rear. The ceramic element is made up of a series of ceramic discs or stacked cylinders. These discs are made in such a way that when excited by a voltage they will expand longitudinally. The discs are wired in parallel and expand in phase. The front mass is a steel cylinder used as a piston. When a sinusiodal voltage is applied to a ceramic element, the piston creates the sound waves by compressing the water in front of the piston. The piston must be of a stiff material so that it does not deform during the compression cycle.

The transducer can be represented in an electrical sense as an equivalent circuit. Such a general equivalent circuit is shown in Fig. 1-2. Such a circuit is made up of three major parts: the source impedance, the mechanical impedance, and the load impedance. The source impedance is made up of the electrical impedance coming from the electrical source and the transmission lines to the transducer. The mechanical impedance has to do with the actuation of the transducer. Such items as the compliance of the ceramic element, the masses of the front and back mass, and the power losses expended in oscillating the front mass are included in this category. The load impedance is the impedance of the medium to the production of sound waves.

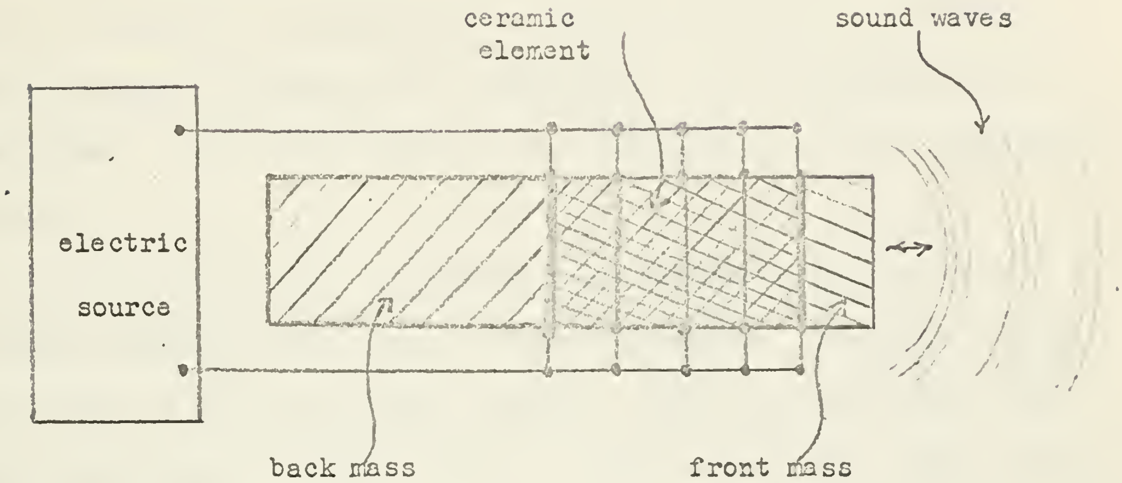


Fig. 1-1. Major Elements of a Transducer.

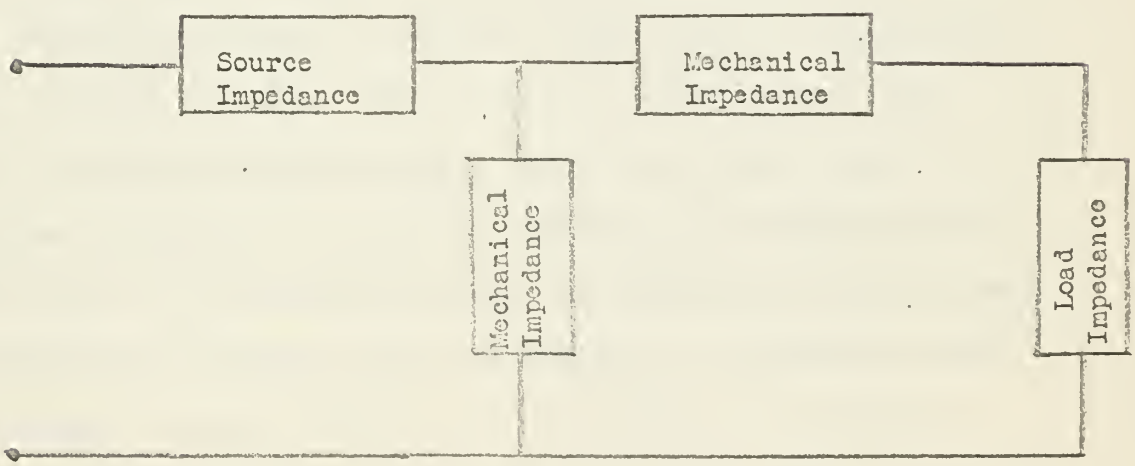


Fig. 1-2. Transducer Equivalent Circuit.

The purpose of this paper is to analyze the impedance characteristics of a transducer and observe the effect of a coupling device to improve the impedance characteristics. Related to this, the frequency response, resonant frequency, and directivity of the transducer will be investigated. The coupling device to be investigated will be a mathematical horn.

It is well known that for maximum power transfer, the source and mechanical impedance should equal the load impedance. In actual circumstances, this is an area of great mismatch. With the use of small area pistons, the resistive loading effect in water is very low. Much of the power radiated into water does not result in the production of sound waves. The theory section will show that such a coupling device as the horn should increase to some extent the resistive loading into the water and therefore, the real power transfer into the water.

There are two opposing factors encountered in transducer design. One is the size of the transducer, the other is the frequency of transmission. In order to produce an effective search beam, the transducers are usually mounted in arrays. The transducers are fired in a timed sequence to obtain the desired results. This factor limits the size of the individual transducers in order that the array can be realistically carried by a ship. This necessitates that the pistons be of small diameter. For realistic transducer sizes, the frequency of transmission with ceramic elements is high (5-100 kc.). High frequency transmission in water is rapidly attenuated as can be seen in Fig. 1-3. Since the piston diameter must be at least one quarter wavelength of the frequency of transmission to radiate this frequency into the water; piston size would have to be increased in order to lower the frequency of transmission.

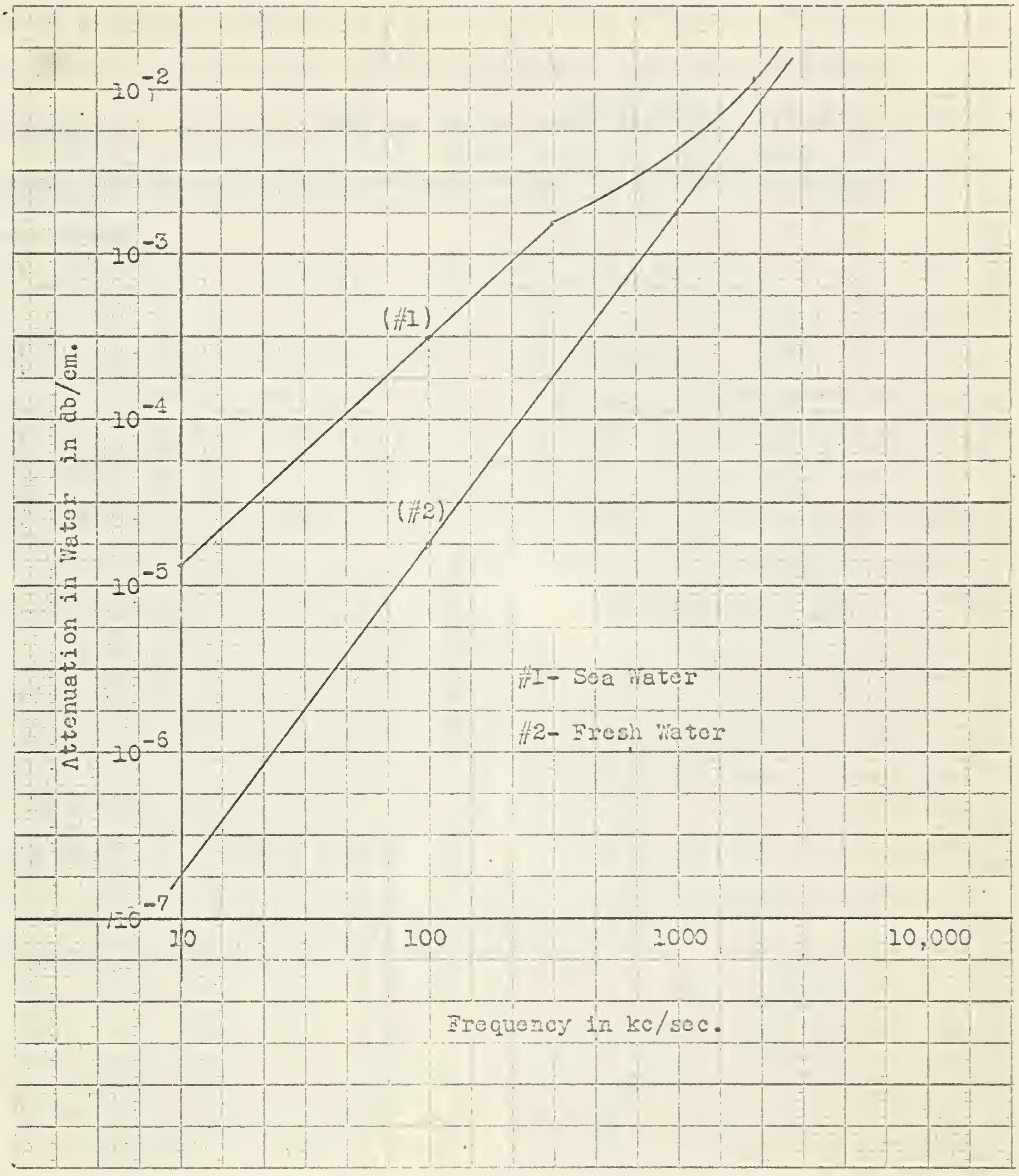


Fig. 1-3. Attenuation of Sound Waves in Water

Another possible advantage of a coupling device therefore, would be to increase the effective size of the piston and allow lower frequency transmission. Also this coupling device could have some effect on the resonant frequency of the transducer and possibly make it a broader band device.

CHAPTER II

THE GENERAL WAVE EQUATION [1, 3]

Sound energy is propagated through water in the form of waves. The type of wave can be either simple or very complex depending on the condition of the sea, the physical arrangement of the transducers, resonant frequencies, bottom and top reflection, thermoclines, salinity, turbulence and several other factors. Before attempting to define the type of wave action applying to the work done on this project, the general acoustic wave equation will be developed. The following symbols will be used:

x, y, z — coordinates of the particle in the medium

ε, η, ζ — component particle displacements along x, y, z and z respectively

u, v, w — component particle velocities along x, y and z respectively

ρ' — instantaneous density at any point

ρ — constant mean density at any point

s — condensation at any point

$$s = \frac{\rho' - \rho}{\rho}$$

p' — instantaneous pressure at any point

p_0 — constant mean pressure at any point

p — excess pressure at any point

$$p = p' - p_0$$

ϕ — velocity potential

c — velocity of propagation of the wave

The term "particle" of the medium is defined as any volume element of the medium which is small enough so that acoustic variables such as pressure and velocity may be considered as constant throughout the volume.

The following assumptions are necessary to proceed with the development.

(1) The fluid medium is isotropic and homogeneous.

(2) The medium contains no dissipative forces, i.e., the medium is perfectly elastic.

(3) The medium is unbounded.

(4) The wave amplitudes are of relatively small amplitude.

(5) Gravitational forces are so small they can be neglected.

Consider a small rectangular volume dx dy dz (See Fig. 2-1). According to the continuity principle, the rate at which the mass inside the volume increases is equal to the influx rate minus the eflux rate of fluid.

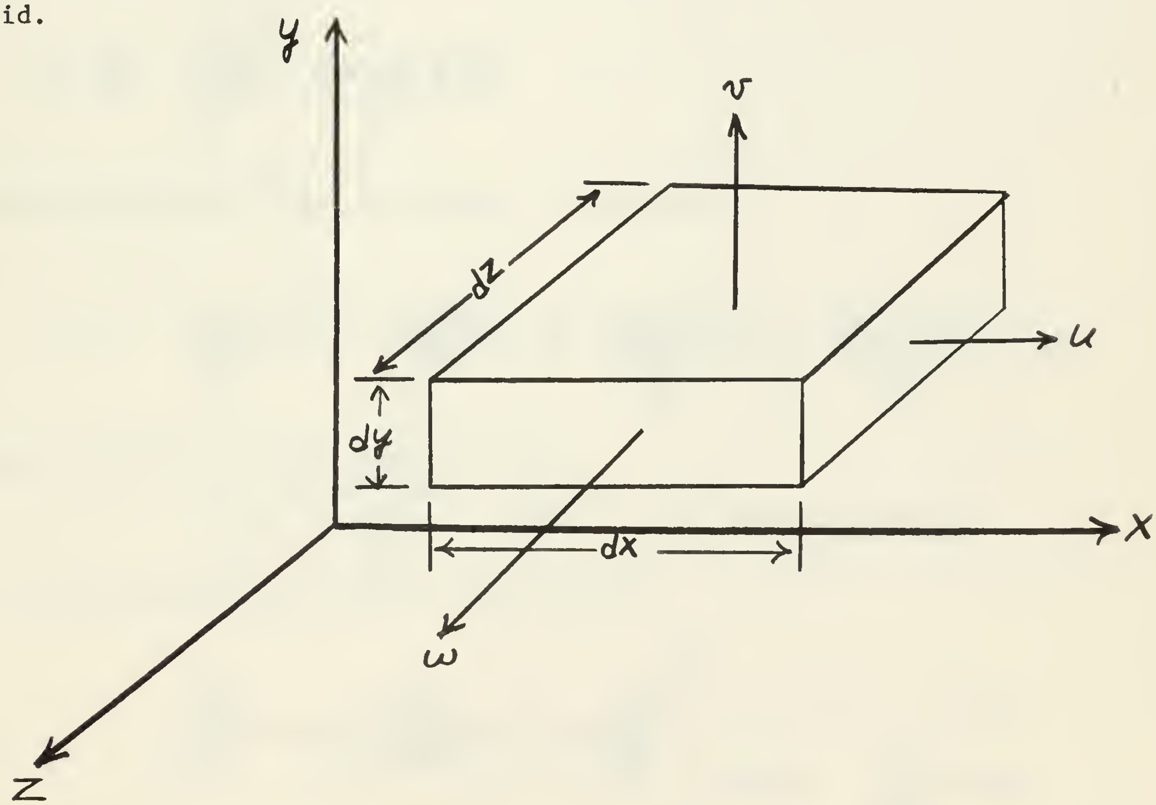


Fig. 2-1 Volume Element in Coordinate System

Considering flow in the x direction and assuming the mass rate is increasing in the volume.

Mass rate of flow out	-	Mass rate of flow in	=	Mass rate increase in volume

Let $\rho' u =$ mass rate of flow out of the volume in the x direction

(2-1) $$\left[\rho' u - \left\{\rho' u + \frac{\partial(\rho' u)}{\partial x}dx\right\}\right]dy\,dz = -\frac{\partial(\rho' u)}{\partial x}dx\,dy\,dz$$

If the same equation is applied to the other two coordinate axis the net change is:

(2-2) $$-\left[\frac{\partial(\rho' u)}{\partial x} + \frac{\partial(\rho' v)}{\partial y} + \frac{\partial(\rho' w)}{\partial z}\right]dx\,dy\,dz$$

The mass rate increase in the volume is:

(2-3) $$\frac{\partial\rho'}{\partial t}dx\,dy\,dz$$

Equating Equation (2-2) and Equation (2-3) gives:

(2-4) $$\frac{\partial\rho'}{\partial t} + \frac{\partial(\rho' u)}{\partial x} + \frac{\partial(\rho' v)}{\partial y} + \frac{\partial(\rho' w)}{\partial z} = 0$$

Using the definition of condensation

$$s = \frac{\rho' - \rho}{\rho} \quad \text{or} \quad \rho' = \rho(1+s)$$

and substituting into Equation (2-3) gives:

$$\frac{\partial\rho'}{\partial t} = \rho\left(\frac{\partial s}{\partial t}\right) + (1+s)\cancel{\frac{\partial\rho}{\partial t}}^{\,0}$$ where ρ is a constant therefore $\frac{\partial\rho}{\partial t} = 0$

Substituting this result into Equation (2-4) and simplifying gives:

$$\rho\frac{\partial s}{\partial t} + \frac{\partial[\rho(1+s)u]}{\partial x} + \frac{\partial[\rho(1+s)v]}{\partial y} + \frac{\partial[\rho(1+s)w]}{\partial z} = 0$$

$$\rho\frac{\partial s}{\partial t} + \rho(1+s)\frac{\partial u}{\partial x} + \rho u\left(\frac{\partial s}{\partial x}\right) + u(1+s)\cancel{\frac{\partial \rho}{\partial x}}^{0} + \rho(1+s)\frac{\partial v}{\partial y}$$

$$+\rho u\frac{\partial s}{\partial y} + v(1+s)\cancel{\frac{\partial \rho}{\partial y}}^{0} + \rho(1+s)\frac{\partial w}{\partial z} + \rho u\left(\frac{\partial s}{\partial z}\right) + w(1+s)\cancel{\frac{\partial \rho}{\partial z}}^{0} = 0$$

(2-5) $$\rho\frac{\partial s}{\partial t} + \rho(1+s)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \rho\left(u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z}\right) = 0$$

In looking at the physical nature of sound waves, it becomes obvious that, for small amplitude disturbances, the instantaneous density is going to change very little from the mean average density when going through a compression and rare-fraction cycle. Therefore the term $s = \frac{\rho' - \rho}{\rho}$ will be extremely small quantity and s compared to unity as in the second term of Equation (2-5) will be insignificant and can be neglected. Therefore, Equation (2-5) simplifies to:

(2-6) $$\rho\frac{\partial s}{\partial t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z}\right) = 0$$

Also since the frequency range examined in this project will not be over 10 KC, the wavelengths are long and therefore; the terms u, v, w, s change very slowly with respect to x, y, z. Thus terms such as u and $\frac{\partial s}{\partial x}$ are small and products of small quantities can be neglected. Therefore:

(2-7) $$\frac{\partial s}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Let $\phi = f(x, y, z, t)$ be known as the velocity potential (a scaler potential). Taking the gradient

(2-8) $$u = \frac{\partial \phi}{\partial x} \; ; \quad v = \frac{\partial \phi}{\partial y} \; ; \quad w = \frac{\partial \phi}{\partial z}$$

substituting into Equation (2-7) gives:

$$\frac{\partial s}{\partial t} + \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

using the LaPlacian operator the equation becomes:

(2-9) $$\frac{\partial s}{\partial t} + \nabla^2 \phi = 0$$

In Equation(2-9) there are still two independent variables. It is therefore necessary to find a means of reducing this to one independent variable namely the velocity potential. This can be done by examining the forces acting on the volume element in Fig. 2-1.

$$F = p A$$

where A is the area of the face of volume

p = pressure on that face

Considering the net force acting on the volume dxdydz in the x direction:

$$dF_x = \left[p' - \left(p' + \frac{\partial p'}{\partial x} dx \right) \right] dy \, dz$$

(2-10) $$dF_x = - \frac{\partial p'}{\partial x} dx \, dy \, dz$$

The time rate of change of momentum is given as:

$$\frac{d(mV)}{dt}$$ where V = velocity

(2-11) $$\frac{d(mV)}{dt} = \frac{\partial(\rho' u \, dx \, dy \, dz)}{\partial t}$$

Using the F = ma formula for force

$$F = m \frac{dV}{dt} \quad \text{or} \quad F = \frac{d(mV)}{dt}$$

So force is equal to the time rate of change of momentum. Equating Equation (2-10) and Equation (2-11);

$$- \frac{\partial p'}{\partial x} dx \, dy \, dz = \frac{\partial(\rho' u \, dx dy dz)}{\partial t}$$

$$(2\text{-}12) \qquad -\frac{\partial p'}{\partial x} = \frac{\partial(\rho' u)}{\partial t}$$

Again using

$$\rho' = \rho(1+s)$$ and neglecting s

in relation to unity gives:

$$\rho' u = \rho u$$

Since

$$\frac{\partial(\rho' u)}{\partial t} = \rho\frac{\partial u}{\partial t} + u\cancelto{0}{\frac{\partial \rho}{\partial t}}$$

therefore Equation (2-12) becomes:

$$\frac{\partial p'}{\partial x} + \rho\frac{\partial u}{\partial t} = 0$$

Again applying this to the other coordinate axis:

$$\frac{\partial p'}{\partial y} + \rho\frac{\partial v}{\partial t} = 0 \; ; \quad \frac{\partial p'}{\partial z} + \rho\frac{\partial w}{\partial t} = 0$$

Relating these terms to Equation (2-10) gives,

$$\left[\frac{\partial p'}{\partial x} + \rho\frac{\partial u}{\partial t}\right]dx + \left[\frac{\partial p'}{\partial y} + \rho\frac{\partial v}{\partial t}\right]dy + \left[\frac{\partial p'}{\partial z} + \rho\frac{\partial w}{\partial t}\right]dz = 0$$

or

$$(2\text{-}13) \qquad \frac{\partial p'}{\partial x}dx + \frac{\partial p'}{\partial y}dy + \frac{\partial p'}{\partial z}dz + \rho\frac{\partial}{\partial t}\left[u\,dx + v\,dy + w\,dz\right] = 0$$

Substituting:

$$u = \frac{\partial \phi}{\partial x} \; ; \quad v = \frac{\partial \phi}{\partial y} \; ; \quad w = \frac{\partial \phi}{\partial z}$$

and

$$dp' = \frac{\partial p'}{\partial x}dx + \frac{\partial p'}{\partial y}dy + \frac{\partial p'}{\partial z}dz$$

into Equation (2-13) gives:

$$dp' + \rho\frac{\partial}{\partial t}\left[\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz\right]$$

or:

$$(2\text{-}14) \qquad dp' + \rho\frac{\partial}{\partial t}\left[d\phi\right] = 0$$

Integrating Equation (2-14):

$$p' + \rho\frac{\partial \phi}{\partial t} = C$$

If this equation is to hold when no acoustic waves are present, the boundary condition ϕ = a constant must apply.

Therefore: $$\frac{\partial \phi}{\partial t} = 0$$

The instantaneous pressure p' is therefore equal to the mean pressure p_0 at the boundary condition.

Substituting in the constant of integration gives $p = p' - p_0$

$$p' + \rho \frac{\partial \phi}{\partial t} = p_0$$

$$p = -\rho \frac{\partial \phi}{\partial t} \qquad (2\text{-}15)$$

It is now necessary to find an equation relating p & s. This is done by considering the bulk modulus of elasticity B.

$$B = \frac{dP}{dV/V} \qquad (2\text{-}16)$$

where: P = incremental pressure

dV/V = strain

$$\frac{dV}{V} = -s \qquad dP = p$$

Substituting into Equation (2-16) gives:

$$p = sB$$

Let

$$c^2 = \frac{B}{\rho}$$

then

$$p = \rho c^2 s$$

Substituting into Equation (2-15) $\rho c^2 s = -\rho \frac{\partial \phi}{\partial t}$

$$s = -\frac{1}{c^2}\left(\frac{\partial \phi}{\partial t}\right)$$

$$\frac{\partial s}{\partial t} = -\frac{1}{c^2}\left(\frac{\partial^2 \phi}{\partial t^2}\right) \qquad (2\text{-}17)$$

Substituting into Equation (2-9) gives:

$$-\frac{1}{c^2}\left(\frac{\partial^2 \phi}{\partial t^2}\right) = -\nabla^2 \phi$$

or

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi \tag{2-18}$$

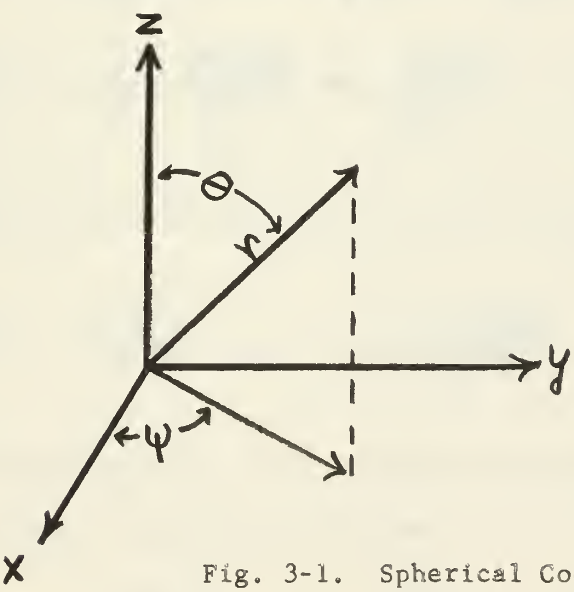
Equation (2-18) is the general three dimentional acoustic wave equation applicable to both liquids and gases. In this form it is expressed as a velocity potential but the other dependent variables could be used with ease and is just a matter of substitution. The general wave equation is the starting point. It is necessary to modify this equation to a form that is applicable to the experimental work. One must ask what "type" of wave best describes the wave propagation from the transducer with and without the horn. This question will be resolved in Chapters III and IV.

CHAPTER III

A. APPLICATION OF WAVE EQUATION TO THE TRANSDUCER WITHOUT HORN

Assume that a transducer is immersed in a great body of water. The piston area is small in comparison to the volume of the fluid. It is logical to assume that a diverging wave pattern will be set up and the small transducer will be transparent to the wave pattern. This wave pattern could be thought of as a sperical acoustic wave pattern. The transducer can be thought of as a point source. In a bounded system with a transducer of dimensional size this idealized situation is corrupted somewhat but is still a good approximation. Directivity patterns made on a single transducer centrally located in a bounded tank seem to support this reasoning. Therefore, the analysis made on a transducer without the horn will be based on spherical wave theory.

Returning to the general wave equation Equation (2-18), it will be most convenient to use this equation in terms of cylindrical coordinates.



where

$$x = r \sin\theta \cos\psi$$
$$y = r \sin\theta \sin\psi$$
$$z = r \cos\theta$$

Fig. 3-1. Spherical Coordinate System

The waves have spherical symetry therefore ϕ is a function of radius and time only.

Taking the LaPlacian operator:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

and transforming it into spherical coordinates:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2\partial\phi}{r d r} + \frac{1}{r^2 \sin\theta} \cdot \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\psi} \cdot \frac{\partial^2\phi}{\partial\psi^2} \tag{3-1}$$

Because the waves have spherical symmetry; Equation (3-1) can be simplified:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2\partial\phi}{r\partial r} \tag{3-2}$$

Differentiating $r\phi$ as a product:

$$\frac{\partial(r\phi)}{\partial r} = r\frac{\partial\phi}{\partial r} + \phi$$

$$\frac{\partial^2(r\phi)}{\partial r^2} = r\frac{\partial^2\phi}{\partial r^2} + \frac{\partial\phi}{\partial r} + \frac{\partial\phi}{\partial r}$$

$$\frac{1}{r} \cdot \frac{\partial^2(r\phi)}{\partial r^2} = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial\phi}{\partial r} \tag{3-3}$$

Substituting Equation (3-3) into Equation (3-2) gives:

$$\nabla^2\phi = \frac{1}{r} \cdot \frac{\partial^2(r\phi)}{\partial r^2} \tag{3-4}$$

Now differentiating $r\phi$ as a product with respect to t and also noting that r is not a function of t gives :

$$\frac{\partial(r\phi)}{\partial t} = r\frac{\partial\phi}{\partial t} + \phi \cancelto{0}{\frac{\partial r}{\partial t}}$$

$$\frac{\partial^2(r\phi)}{\partial t^2} = r\frac{\partial^2\phi}{\partial t^2} + \frac{\partial\phi}{\partial t}\cdot\cancel{\frac{\partial r}{\partial t}}^{\,0}$$

$$\frac{\partial^2\phi}{\partial t^2} = \frac{1}{r}\cdot\frac{\partial^2(r\phi)}{\partial t^2} \tag{3-5}$$

Substituting Equation (3-4) and Equation (3-5) into the gèneral acoustic wave Equation (2-18) gives:

$$\frac{\partial^2(r\phi)}{\partial t^2} = c^2\frac{\partial^2(r\phi)}{\partial r^2} \tag{3-6}$$

If the product $(r\emptyset)$ is considered as a single variable, Equation (3-6) is nothing more than an ordinary second order partial differential equation. The general solution is:

$$r\phi = f_1(ct-r) + f_2(ct+r) \tag{3-7}$$

This solution contains two terms. The first one describes a spherical diverging wave; the second term describes a spherical converging wave. This then, is the general solution of a spherical wave starting or originating from a large sphere of radius r. Outside the sphere a diverging wave is traveling outward and inside the sphere a converging wave is traveling toward the origin.

In the experimental work done in this paper, the waves emanate from a small source (can be thought of as a point source) therefore, the converging wave is of no importance. The only part of the solution that need be considered is the diverging wave.

The equation then, for a diverging spherical wave whose vibrations are harmonically expressed in complex notation is:

$$r\phi = \vec{A}e^{j(\omega t - \beta r)} \tag{3-8}$$

where β is the wave number

w = natural frequency

$\vec{A}$ = wave amplitude (possibly complex)

B. SPECIFIC ACOUSTIC IMPEDANCE FOR THE TRANSDUCER WITHOUT HORN

The main purpose of developing the wave theory has been to define and be able to use it. The specific acoustic impedance is the impedance of the medium to a disturbance passing through the medium. As in mechanical systems this impedance is a ratio of pressure to velocity.[4]

$$z = \frac{p}{u} \tag{3-9}$$

where z = specific acoustic impedance

This wave equation can be expressed in terms of pressure and velocity by referring to Equation (2-15) and Equation (2-8) used in the derivation of the general wave equation.

$$p = -\rho \frac{\partial \phi}{\partial t} \tag{2-15}$$

$$u = \frac{\partial \phi}{\partial x} \tag{2-8}$$

or in this case

$$u = \frac{\partial \phi}{\partial r} \tag{3-10}$$

Substituting the Equation (3-8) into Equation (2-15) and (3-9) gives:

$$p = -j \rho \omega \frac{\vec{A}}{r} e^{j(\omega t - \beta r)} \tag{3-11}$$

$$u = \frac{\vec{A}}{r} e^{j(\omega t - \beta r)} \left[-j\beta - \frac{1}{r} \right] \tag{3-12}$$

Taking the ratio of p/u gives:

$$\frac{p}{u} = \frac{-j\rho\omega \frac{\vec{A}}{r} e^{j(\omega t - \beta r)}}{\left[-j\beta - \frac{1}{r}\right]\frac{\vec{A}}{r} e^{j(\omega t - \beta r)}} = \frac{-j\rho\omega}{-j\beta\left(1 - \frac{j}{\beta r}\right)}$$

$$= \frac{\rho\omega}{\beta\left(1 - \frac{j}{\beta r}\right)} = \frac{\rho\omega\left(1 + \frac{j}{\beta r}\right)}{\beta\left(1 - \frac{j}{\beta r}\right)\left(1 + \frac{j}{\beta r}\right)} = \frac{\rho\omega\left(\frac{\beta r + j}{\beta r}\right)}{\beta\left(1 + \frac{1}{\beta^2 r^2}\right)}$$

$$\text{(3-13)} \quad \frac{p}{u} = \frac{\rho\omega(\beta r + j)}{\frac{\beta(\beta^2 r^2 + 1)}{\beta r}} = \frac{\rho\omega r(\beta r + j)}{1 + \beta^2 r^2}$$

Eliminating w:

$$\omega = 2\pi f = \frac{2\pi c}{\lambda}$$

The wave number is defined as

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

Substituting into Equation (3-13):

$$z = \frac{p}{u} = \frac{\rho c \beta r(\beta r + j)}{1 + \beta^2 r^2}$$

$$\text{(3-14)} \quad z = \rho c \frac{\beta^2 r^2}{1 + \beta^2 r^2} + j\rho c \frac{\beta r}{1 + \beta^2 r^2}$$

As can be seen from Equation (3-14), the specific acoustic impedance is a function of the frequency and radial distance from the source. These functions are analyzed for several distances from the source over the frequency spectrum. Figs. 3-2, 3-3 and 3-4 show the specific acoustic impedance for several locations in the medium.

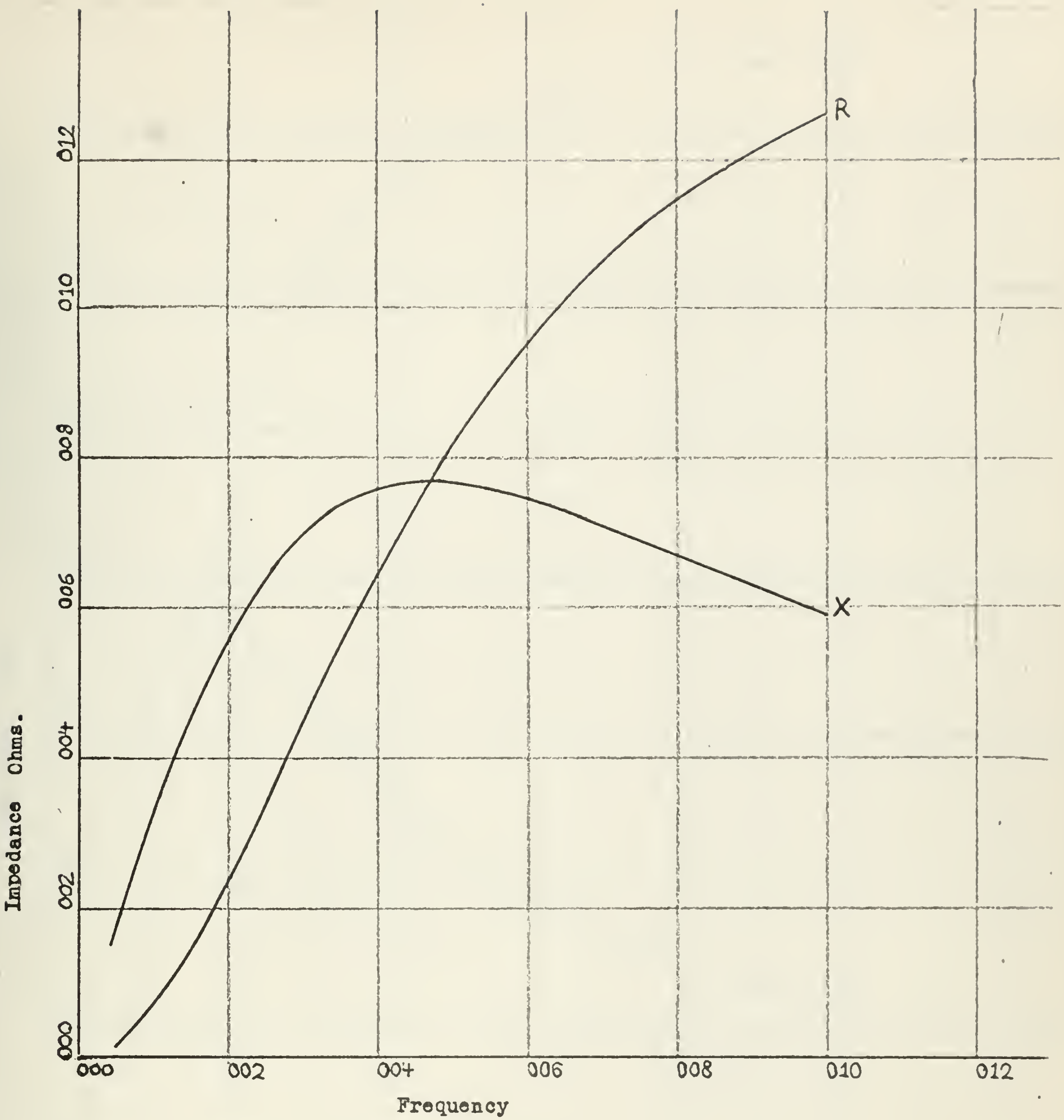


Fig. 3-2 Transducor - No Horn.

X-SCALE = 2.00E+03 UNITS/INCH.
Y-SCALE = 2.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY
D=2IN.

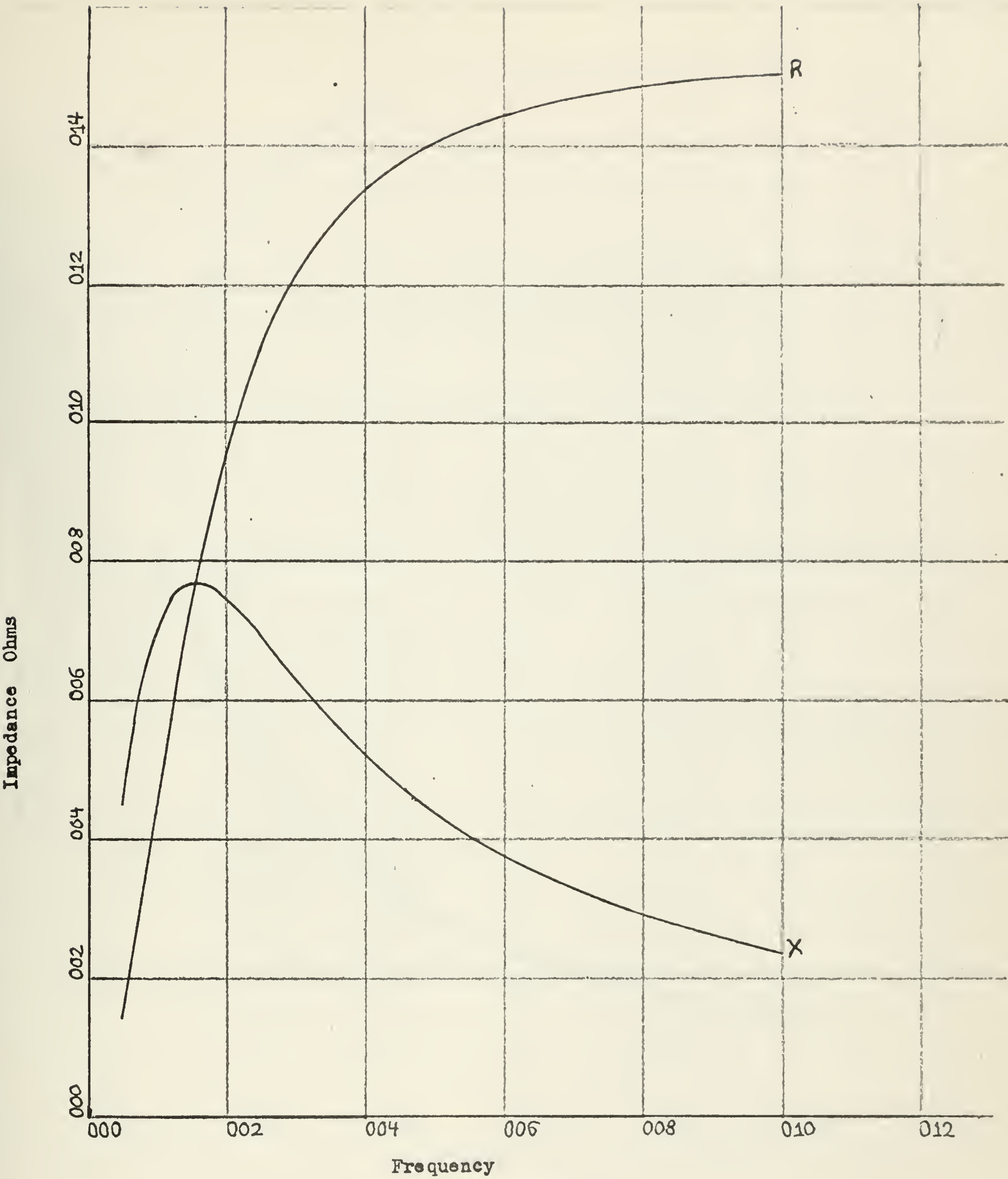


Fig. 3-3 Transducer - No Horn

X-SCALE = 2.00E+03 UNITS/INCH.
Y-SCALE = 2.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY
D=6IN.

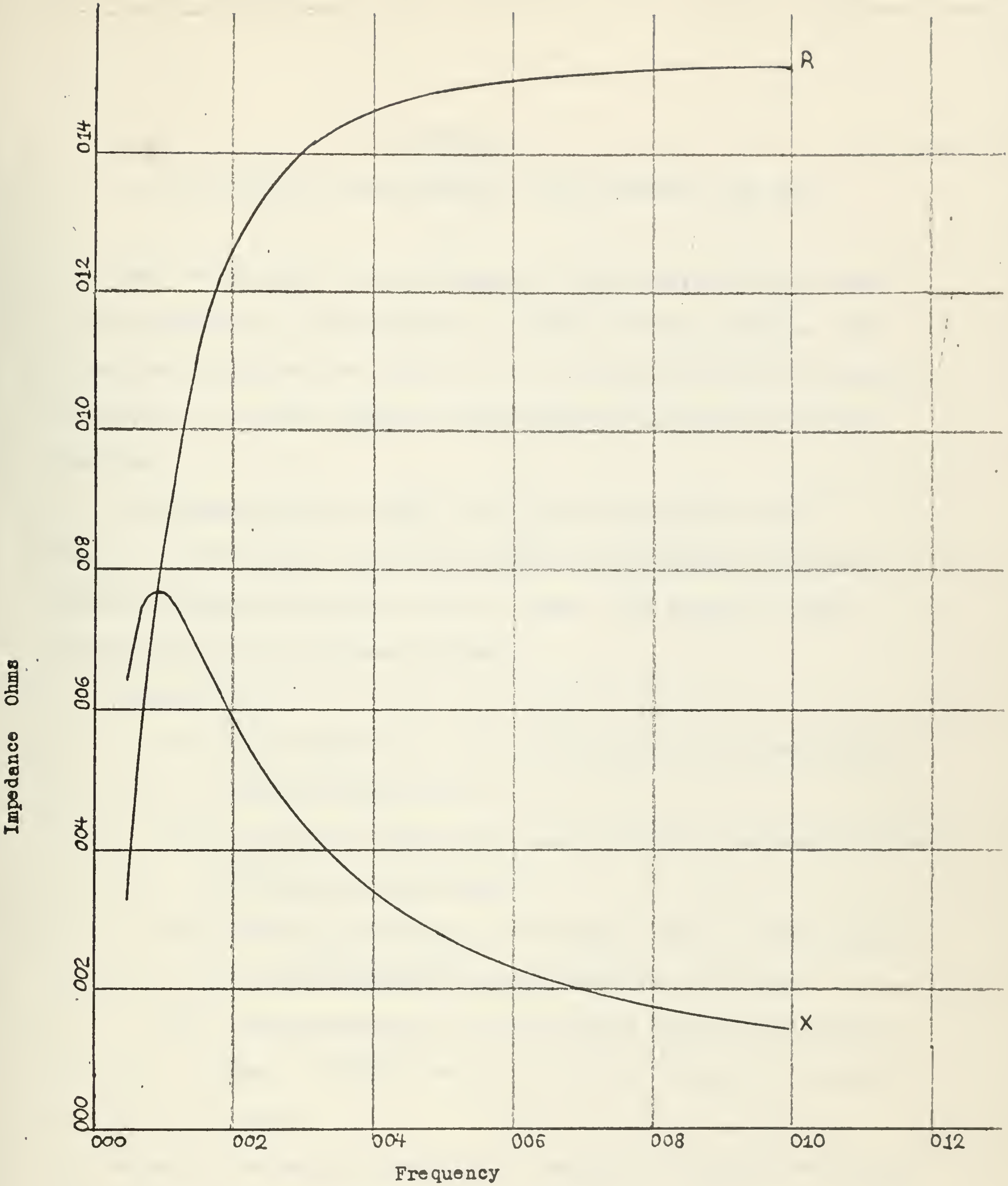


Fig. 3-4 Transducer - No Horn

X-SCALE = 2.00E+03 UNITS/INCH.
Y-SCALE = 2.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY
D=10IN.

CHAPTER IV

A. APPLICATION OF WAVE EQUATION TO THE TRANSDUCER WITH HORN

This chapter deals with the analysis of the mathematical horn used in the experiment. The development is similar to that of Chapter III; first the appropriate wave equation will be developed and then the associated specific acoustic impedance of the medium within the horn will be analyzed.

The mathematical horn used in this project is called a $\cosh^2$ horn. In order to get a usable mathematical expression for the horn several stringent assumptions have to be made. The effects of these assumptions will be discussed in Chapter V.

Assumptions:

(1) All assumptions and restrictions made for the derivations are still applicable.

(2) It will be assumed that acoustic energy is propagated through the horn as plane waves.

(3) The walls of the horn are infinitly stiff, so that there is no loss transversly through the walls of the horn.

(4) The plane waves do not lose contact with the walls of the horn. In other words, the flare of the horn is not too great.

Using the continuity principle as in Chapter II, consider the volume element shown in Fig. (4-1).

Since plane waves are being considered only the x direction is pertinent.

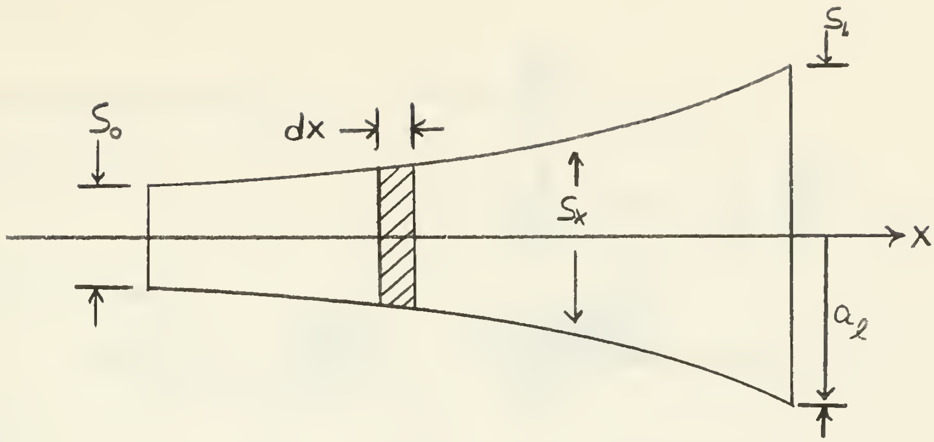


Fig. (4-1) Volume element of the Horn

Rate of mass increase:

$$\left[\rho' u - \left\{\rho' u + \frac{\partial(\rho' u S_x)}{\partial x} dx\right\}\right] = \frac{\partial(\rho' u S_x)}{\partial x} dx$$

Since the instantaneous density at any point is numerically close to the mean constant density

(4-1) $$\frac{\partial(\rho' S_x u)}{\partial x} dx \approx -\rho \frac{\partial(S_x u)}{\partial x} dx$$

The rate of mass increase would necessarily be:

(4-2) $$\frac{\partial \rho'}{\partial t} S_x dx \approx \rho S_x \frac{\partial s}{\partial t} dx$$

Since $$\frac{\partial s}{\partial t} = \frac{\rho \frac{\partial \rho'}{\partial t} - \rho' \cancel{\frac{\partial \rho}{\partial t}}^{0}}{\rho^2} = \frac{\frac{\partial \rho'}{\partial t}}{\rho}$$

Therefore: $$\frac{\partial \rho'}{\partial t} = \rho \frac{\partial s}{\partial t}$$

Equating Equation (4-1) and Equation (4-2) gives:

$$\rho S_x \frac{\partial s}{\partial t} dx = -\rho \frac{\partial(S_x u)}{\partial x} dx$$

(4-3) $$S_x \frac{\partial s}{\partial t} + \frac{\partial(S_x u)}{\partial x} = 0$$

Substituting Equation (2-8) $\quad u = \frac{\partial \phi}{\partial x}$

And Equation (2-14)

$$S = \frac{p}{\rho c^2} = -\frac{1}{c^2} \cdot \frac{\partial \phi}{\partial t}$$

into the (4-3) gives:

$$S_x\left[-\frac{1}{c^2} \cdot \frac{\partial^2 \phi}{\partial t^2}\right] + \frac{\partial\left[S_x\left(\frac{\partial \phi}{\partial x}\right)\right]}{\partial x} = 0$$

Simplifying:

$$-\frac{S_x}{c^2} \cdot \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial S_x}{\partial x} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial^2 \phi}{\partial x^2}(S_x)$$

$$\frac{\partial^2 \phi}{\partial t^2} = c^2\left[\frac{\frac{\partial S_x}{S_x}}{\partial x} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial^2 \phi}{\partial x^2}\right]$$

(4-4) $$\frac{\partial^2 \phi}{\partial t^2} = c^2\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial x} \cdot \frac{\partial(\ln S_x)}{\partial x}\right]$$

Equation (4-4) shows that the plane wave equation is modified by the addition of the horn. The modified plane wave equation contains an additional term which is a function of ϕ and the crossectional area of the horn. This shows that the plane wave equation and therefore the specific acoustic impedance will change depending on the type of horn.

The type of horn to be used is described by the Equation (4-5).

(4-5) $$S_x = S_o \cosh^2\left(\frac{x}{9}\right)$$

The following steps show the operations necessary to fit this Equation into Equation (4-4):

$$\cosh\left(\frac{2x}{h}\right) = 2\cosh^2\left(\frac{x}{h}\right) - 1$$

$$\cosh^2\left(\frac{x}{h}\right) = \frac{\cosh\left(\frac{2x}{h}\right) - 1}{2}$$

$$S_x = \frac{S_o}{2}\left[\cos\left(\frac{2x}{h}\right) + 1\right]$$

$$\ln S_x = \ln \frac{S_o}{2} + \ln\left[\cos\frac{2x}{h} + 1\right]$$

$$\frac{d \ln S_x}{dx} = \cancel{\frac{d \ln \frac{S_o}{2}}{dx}} \nearrow 0 + \frac{d \ln\left[\cos\left(\frac{2x}{h}\right) + 1\right]}{dx}$$

$$\frac{d \ln S_x}{dx} = \frac{1}{\cos\left(\frac{2x}{h}\right) + 1}\left[-\sinh\left(\frac{2x}{h}\right) \cdot \frac{2}{h}\right]$$

(4-6) $$\frac{d \ln S_x}{dx} = -\frac{2}{h}\left[\frac{\sinh\left(\frac{2x}{h}\right)}{\cosh\left(\frac{2x}{h}\right) + 1}\right]$$

The wave equation for this horn is then:

(4-7) $$\frac{\partial^2 \phi}{\partial t^2} = c^2\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial x} \cdot -\frac{2}{h}\left(\frac{\sinh\left(\frac{2x}{h}\right)}{\cosh\left(\frac{2x}{h}\right) + 1}\right)\right]$$

This partial differential equation cannot be solved mathematically because of the complicated expression that has been added. Therefore, it is necessary to find a compromise that will approximately describe this horn and can be solved mathematically. A logical choice would be to try and fit this horn to an exponential horn. The flare of the $\cosh^2$ horn is plotted on semi-log paper (See Fig. 4-2), and approximated by the

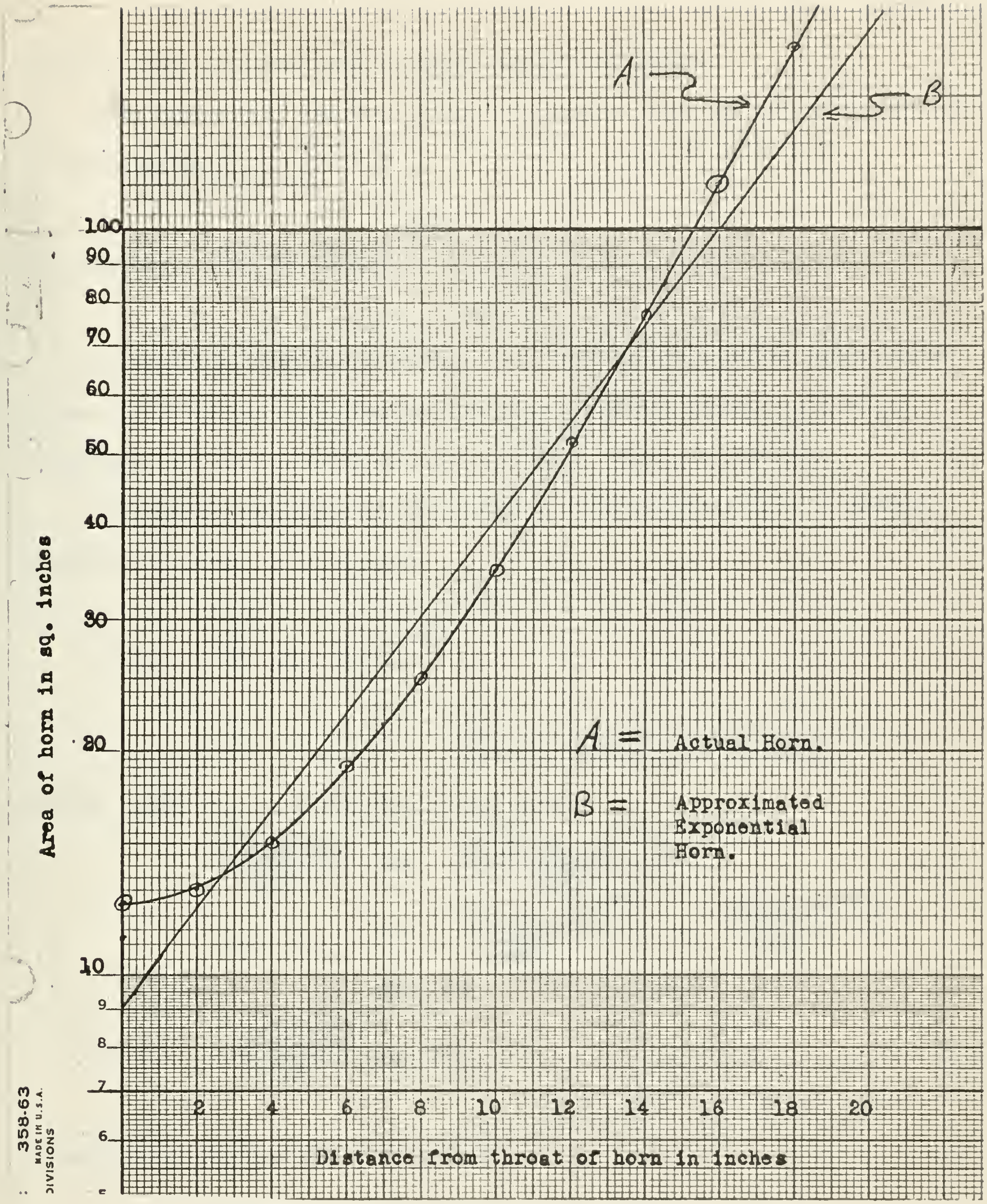


Fig. 4-2. Graphical Layout of Mathamatical Horn

best straight line that can be drawn through the points.

From Figure 4-2, the best single exponential horn for the approximation is shown to be:

$$S_x = 9.0\, e^{.15x} \tag{4-8}$$

It should be observed from Fig. 4-2, that two exponential horns would give a still better approximation but this would add another reflection interface into the calculations and seriously complicate matters.

In order to put Equation (4-4) into a more practical form, a general equation for S_x will be used.

$$S_x = S_o\, e^{mx} \tag{4-9}$$

where m is the flare constant.

Taking $\ln S_x$ and differentiating gives:

$$\frac{\partial(\ln S_x)}{\partial x} = \frac{\partial(\ln S_o\, e^{mx})}{\partial x}$$

$$= \cancel{\frac{\partial(\ln S_o)}{\partial x}}^{\,0} + \frac{\partial(\ln e^{mx})}{\partial x}$$

$$= m$$

$$\frac{\partial(\ln S_x)}{\partial x} = m \tag{4-10}$$

Substituting in Equation (4-4) gives:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \left[\frac{\partial^2 \phi}{\partial x^2} + m \frac{\partial \phi}{\partial x} \right] \tag{4-11}$$

The general solution of this partical differential equation is:

(4-12) $$\vec{\phi} = e^{-\alpha x}\left(\vec{A}\, e^{j(\omega t - \beta x)} + \vec{B}\, e^{j(\omega t + \beta x)}\right)$$

Differentiating Equation (4-12) to find the constants α and β :

$$\frac{\partial \phi}{\partial x} = e^{-\alpha x}\left[\vec{A}\, e^{j(\omega t - \beta x)} \cdot -j\beta + \vec{B}\, e^{j(\omega t + \beta x)} \cdot j\beta\right]$$

$$+ \left[\vec{A}\, e^{j(\omega t - \beta x)} + \vec{B}\, e^{j(\omega t + \beta x)}\right] \cdot -\alpha e^{-\alpha x}$$

Let $$(A + B) = e^{-\alpha x}\left[\vec{A}\, e^{j(\omega t - \beta x)} + \vec{B}\, e^{j(\omega t + \beta x)}\right]$$

then: $$\frac{\partial \phi}{\partial x} = +j\beta(-A+B) - \alpha(A+B)$$

$$\frac{\partial^2 \phi}{\partial x^2} = j^2\beta^2(A+B) - 2j\alpha\beta(-A+B) + \alpha^2(A+B)$$

$$\frac{\partial \phi}{\partial t} = j\omega(A+B)$$

$$\frac{\partial^2 \phi}{\partial t^2} = j^2\omega^2(A+B)$$

Substituting into Equation (4-11):

$$j^2\omega^2(A+B) = c^2 j^2 \beta^2(A+B) - 2j\alpha\beta c^2(-A+B) + \alpha^2 c^2(A+B)$$

$$+ mc^2 j\beta(-A+B) - mc^2\alpha(A+B)$$

Equating like terms:

$$j^2\omega^2(A+B) = c^2 j^2 \beta^2(A+B) + \alpha^2 c^2(A+B)$$

$$- mc^2\alpha(A+B)$$

$$m = 2\alpha$$

$$j^2 = -1$$

$$\frac{\omega^2}{c^2} = k^2$$

$$-k^2 = -\beta^2 + \alpha^2 - 2\alpha^2$$

$$\beta^2 = k^2 - \frac{m^2}{4} \quad ; \quad \beta = \sqrt{k^2 - \frac{m^2}{4}}$$

Therefore the constants of integration are:

$$\alpha = \frac{m}{2} \tag{4-13}$$

$$\beta = \sqrt{k^2 - \frac{m^2}{4}} \tag{4-14}$$

Substituting Equation (4-13) and Equation (4-14) into Equation (4-12) gives the general solution for a plane wave treatment of the medium inside the horn.

B. SPECIFIC ACOUSTIC IMPEDANCE FOR THE TRANSDUCER WITH HORN

Using the general harmonic solution (Equation (4-12) and substituting into the specific impedance formula gives:

Let $$A+B = \left[\vec{A}e^{j(\omega t-\beta x)} + \vec{B}e^{j(\omega t+\beta x)}\right]$$

$$\vec{Z} = \frac{p}{u} = \frac{-\rho\frac{\partial\phi}{\partial t}}{\frac{\partial\phi}{\partial x}}$$

$$\vec{Z} = \frac{-\rho j\omega e^{-\alpha x}(A+B)}{-j\beta e^{-\alpha x}(A-B)-\alpha e^{-\alpha x}(A+B)}$$

$$= \frac{\rho j\omega(\vec{A}e^{-j\beta x}+\vec{B}e^{j\beta x})}{\alpha(\vec{A}e^{-j\beta x}+\vec{B}e^{j\beta x})+j\beta(\vec{A}e^{-j\beta x}-\vec{B}e^{j\beta x})}$$

$$= \frac{\rho c k(\vec{A}e^{-j\beta x}+\vec{B}e^{j\beta x})}{-j\left[(\alpha+j\beta)\vec{A}e^{-j\beta x}-(-\alpha+j\beta)\vec{B}e^{j\beta x}\right]}$$

$$= \frac{\rho c k(\vec{A}e^{-j\beta x}+\vec{B}e^{j\beta x})}{(\beta-j\alpha)\vec{A}e^{-j\beta x}-(\beta+j\alpha)\vec{B}e^{j\beta x}}$$

(4-15) $$\vec{Z} = \rho c\left[\frac{\vec{A}e^{-j\beta x}+\vec{B}e^{j\beta x}}{\left(\frac{\beta}{k}-j\frac{\alpha}{k}\right)\vec{A}e^{-j\beta x}-\left(\frac{\beta}{k}+j\frac{\alpha}{k}\right)Be^{+j\beta x}}\right]$$

This is the general expression for the specific acoustic impedance at any point in an exponential horn.

In this expression, the complex quantities $\vec{A}$ and $\vec{B}$ do not cancel out,

therefore it is necessary to examine these quantities more closely. In an infinitely long exponential horn, there would be no reflected wave back into the horn and therefore $\vec{B}$ would be zero. However, in a finite exponential horn there is a possibility of mismatch at the mouth of the horn and a portion of the incident wave would be reflected back. A general rule of thumb used to determine if an exponential horn can be considered infinite stipulates that [3]

$$k a_\ell > 5 \qquad a_\ell = \text{radius of horn at mouth}$$

$$k = \text{wave number}$$

$$k = \frac{\omega}{c}$$

Let

$$f = 1000 \text{ cps}$$

$$c \text{ (in sea water)} = 1504 \text{ m/sec.}$$

$$a_\ell = 9 \text{ in.}$$

$$k a_\ell = \left[\frac{(2\pi)(1000)}{1504}\right]\left[\frac{(9)(2.54)}{100}\right] = .955$$

Since this stipulation does not hold for the horn under study, it is necessary to evaluate $\vec{B}$. It is interesting to note that if the medium to be considered were air, $k a_\ell$ would then be approximately equal to five and an infinite horn could be assumed. This means that for the same horn there is much more reflection or impedance mismatch at the mouth of the horn in sea water than in air. This fact reduces the effectiveness of the horn in water.

In order to determine the value of $\vec{B}$, the impedance at the mouth of the horn must be analyzed. It is a valid assumption to state that ϕ must be continuous at the boundaries - in this case, at the mouth of the horn.

Therefore the specific acoustic impedance at the mouth must be equal to the external loading impedance on the mouth of the horn. Solving Equation (4-15) for $x = \ell$ and equating to an external load impedance gives.

$$\vec{Z}_\ell = \rho c \left[\frac{\vec{A} e^{-j\beta\ell} + \vec{B} e^{j\beta\ell}}{\left(\frac{\beta}{k} - j\frac{\alpha}{k}\right)\vec{A} e^{-j\beta\ell} - \left(\frac{\beta}{k} + j\frac{\alpha}{k}\right)\vec{B} e^{j\beta\ell}} \right]$$ where $\vec{Z}_\ell$ = external load impedance.

Solving for B:

$$\vec{B} e^{j\beta\ell}\left[\rho c + Z_\ell\left(\frac{\beta}{k} + j\frac{\alpha}{k}\right)\right] = \vec{A} e^{-j\beta\ell}\left[\vec{Z}_\ell\left(\frac{\beta}{k} - j\frac{\alpha}{k}\right) - \rho c\right]$$

$$(4\text{-}16) \quad \vec{B} = \frac{\vec{A} e^{-2j\beta\ell}\left(\vec{Z}_\ell\left(\frac{\beta}{k} - j\frac{\alpha}{k}\right) - \rho c\right)}{\rho c + \vec{Z}_\ell\left(\frac{\beta}{k} + j\frac{\alpha}{k}\right)}$$

At this point it is necessary to evaluate $\vec{Z}_\ell$. Based on the assumptions made in the development of the horn wave equations, a volume of the medium at the mouth of the horn can be thought of as a vibrating piston. This will be only approximately true because of frictional effects between the layer of water and the sides of the horn.

Using this idea of a piston at the mouth of the horn, the analogy can be made that this mechanism is a large transducer with a piston area equal to the area of the mouth of the horn S_ℓ. When the transducer without the horn was considered, it was assumed that this transducer could be represented by a spherical point source because of the small dimension of the piston radius in relation to the wave-length. Because of the much larger piston radius of the transducer with horn, this can no longer be considered a spherical point source and it must be assumed that the waves have some directivity. This statement is borne out by the experimental directivity plots made on the transducer with horn. It is therefore necessary to extend the analysis to consider radiation from a piston source.

Starting with the development of the spherical source, let the velocity be described by a sinusoidal pulsating sphere as:

$$u = U_o \cos \omega t$$

Assuming no cavitation phenomena, the particle velocity of the acoustic wave must equal the velocity of the sphere.

Since the specific acoustic impedance has previously been defined as:

$$\vec{Z} = \frac{p}{u}$$

then acoustic wave velocity in terms of z is:

$$\vec{u} = \frac{\vec{A}}{r\vec{Z}} e^{j(\omega t - kr)} \tag{4-17}$$

Therefore setting the velocities equal and evaluating A gives:

$$\frac{\vec{A}}{a\vec{Z}_a} e^{j(\omega t - kr)} = U_o e^{j\omega t}$$

$$\vec{A} = a U_o \vec{Z}_a e^{jkr}$$

$$= r^2 U_o \rho c k \frac{(kr + j)}{1 + k^2 r^2} \left[\cos(kr) + j \sin(kr)\right]$$

Assuming kr is small:

$$\vec{A} = \frac{r^2 U_o \rho c k (kr + j)(1 + jkr)}{1 + k^2 r^2}$$

Substituting A into Equation (3-10)

$$P = j\rho c k r U_o e^{j(\omega t - kr)}$$

Let $Q = 4\pi r^2 U_o$

$$P = \frac{j\rho c k}{4\pi r} Q e^{j(\omega t - kr)} \tag{4-18}$$

Using Equation (4-18) as the expression for the pressure of an acoustic wave caused by a piston radiating in water, it is possible to find the force exerted on the radiating surface.[3] Consider an infinitesimal area d s on the piston and d p will be the pressure exerted on the medium at any point d s′ removed from d s. (See Fig. 4-3).

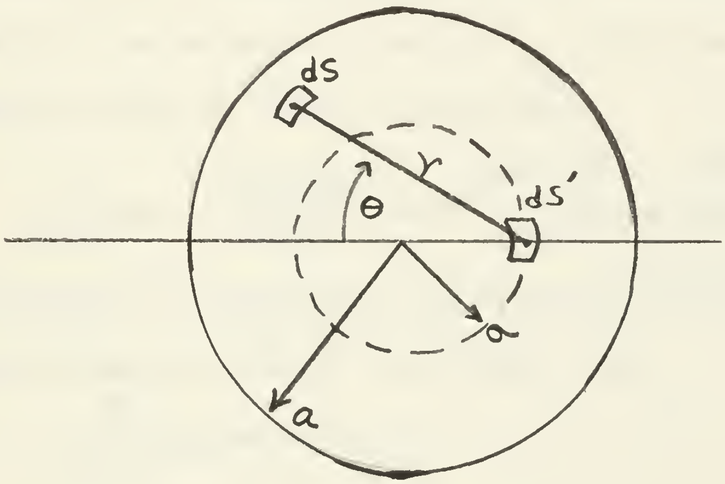


Fig. 4-3 Surface of a Transducer Piston.

By summing over the surface of the piston, the total acoustic pressure at d s can be evaluated.

$$P = \iint \frac{j\rho c k}{4\pi r} u_0 e^{j(\omega t - kr)} \, dS \qquad (4\text{-}19)$$

The reaction force is given by:

$$f_r = -\iint P \, dS'$$

Substituting p from Equation (4-19)

$$f_r = -\frac{j\rho c k}{4\pi} u_0 e^{j\omega t} \iint dS' \iint \frac{e^{-jkr}}{r} \, dS \qquad (4\text{-}20)$$

Referring to the geometry of Fig. 4-3, it is necessary to change the limits of integration to apply to the correct nomenclature. If σ is the radial distance from the center of the piston to element d s and θ is the angle between d s and d s$'$; then

$$r\,d\theta\,dr = ds$$

Also letting d s$'$ $= \sigma\,d\sigma\,d\psi$ where the final integration can be made over the whole piston using ψ from 0 to 2π and σ from 0 to a; Equation (4-20) can be put in the form:

$$f_r = -\frac{j\rho c k}{2\pi} u_0 e^{j\omega t} \int_0^a \sigma\,d\sigma \int_0^{2\pi} d\psi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{2\sigma\cos\theta} e^{-jkr}$$

The integration must be carried out step by step and will necessarily involve Bessel functions because of the integral form:

$$\int_0^{\frac{\pi}{2}} e^{-2jk\sigma\cos\theta}\,d\theta$$

The final result of the force equation is of the form:

$$f_r = -\frac{\rho c \pi a^2}{2} u_0 e^{j\omega t}\left[R_1(2ka) + X_1(2ka)\right] \qquad (4\text{-}21)$$

where $R_1(2ka)$ and $X_1(2ka)$ are the Bessel functions:

$$R_1(x) = \frac{x^2}{2\cdot 4} - \frac{x^4}{2\cdot 4^2\cdot 6} + \frac{x^6}{2\cdot 4^2\cdot 6^2\cdot 8} - \cdots$$

$$X_1(x) = \frac{4}{\pi}\left(\frac{x}{3} - \frac{x^3}{3^2\cdot 5} + \frac{x^5}{3^2\cdot 5^2\cdot 7} - \cdots\right)$$

Use of the concept of radiation impedance can now be used. The

radiation impedance is defined as the ratio of the force exerted by the piston on the medium to the velocity of the piston. Also the radiation impedance is related to the specific acoustic impedance at the surface of the piston by

$$\vec{Z}_r = \frac{-f_r}{u_o e^{j\omega t}} = \frac{\rho c \pi a^2}{2}\left[R_1(2ka) + j X_1(2ka)\right]$$

Substituting a_ℓ for a and Z_ℓ for $\frac{Z_r}{S_\ell}$ gives:

$$\vec{Z}_\ell = \frac{\rho c \pi a^2}{2 S_\ell}\left[R_1(2ka_\ell) + j X_1(2ka_\ell)\right]$$

$$\text{(4-24)} \quad \vec{Z}_\ell = \frac{\rho c}{2}\left[R_1(2ka_\ell) + j X_1(2ka_\ell)\right]$$

By substituting the evaluated values of $\vec{B}$ and $\vec{Z}_\ell$ into Equation (4-15), the generalized expression can be written.

$$\text{(4-25)} \quad \vec{Z} = \rho c \left[\frac{\vec{A} e^{-j\beta x} + \vec{A}\left\{\frac{e^{-2j\beta\ell}\left(\vec{Z}_\ell\left(\frac{\beta}{k} - j\frac{\alpha}{k}\right) - \rho c\right)}{\rho c + \vec{Z}_\ell\left(\frac{\beta}{k} + j\frac{\alpha}{k}\right)}\right\} e^{j\beta x}}{\vec{A}\left(\frac{\beta}{k} - j\frac{\alpha}{k}\right)e^{-j\beta x} - \left(\frac{\beta}{k} + j\frac{\alpha}{k}\right)e^{j\beta x}\vec{A}\left\{\frac{e^{-2j\beta\ell}\left(\vec{Z}_\ell\left(\frac{\beta}{k} - j\frac{\alpha}{k}\right) - \rho c\right)}{\rho c + \vec{Z}_\ell\left(\frac{\beta}{k} + \frac{\alpha}{k}\right)}\right\}}\right]$$

The solution to Equation (4-25) was set up on the digital computer and curves were obtained of impedance v.s. frequency for selected locations within the horn. These curves are shown on Figs. 4-4, 4-5, 4-6, and 4-7.

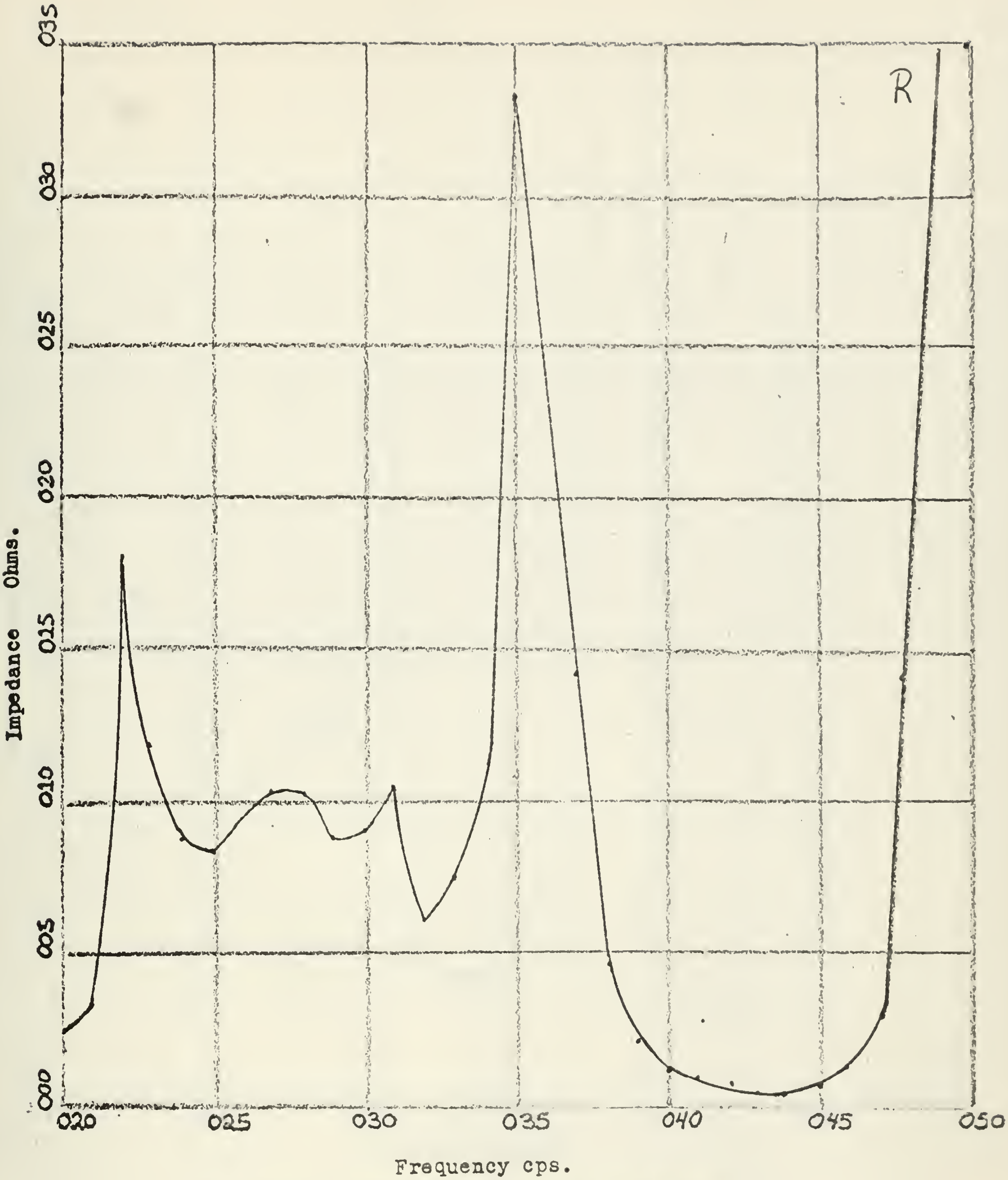


Fig. 4-4. Transducer - With Finitė Horn.

X-SCALE = 5.00E+02 UNITS/INCH.
Y-SCALE = 5.00E+04 UNITS/INCH.
SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY
D=0IN.

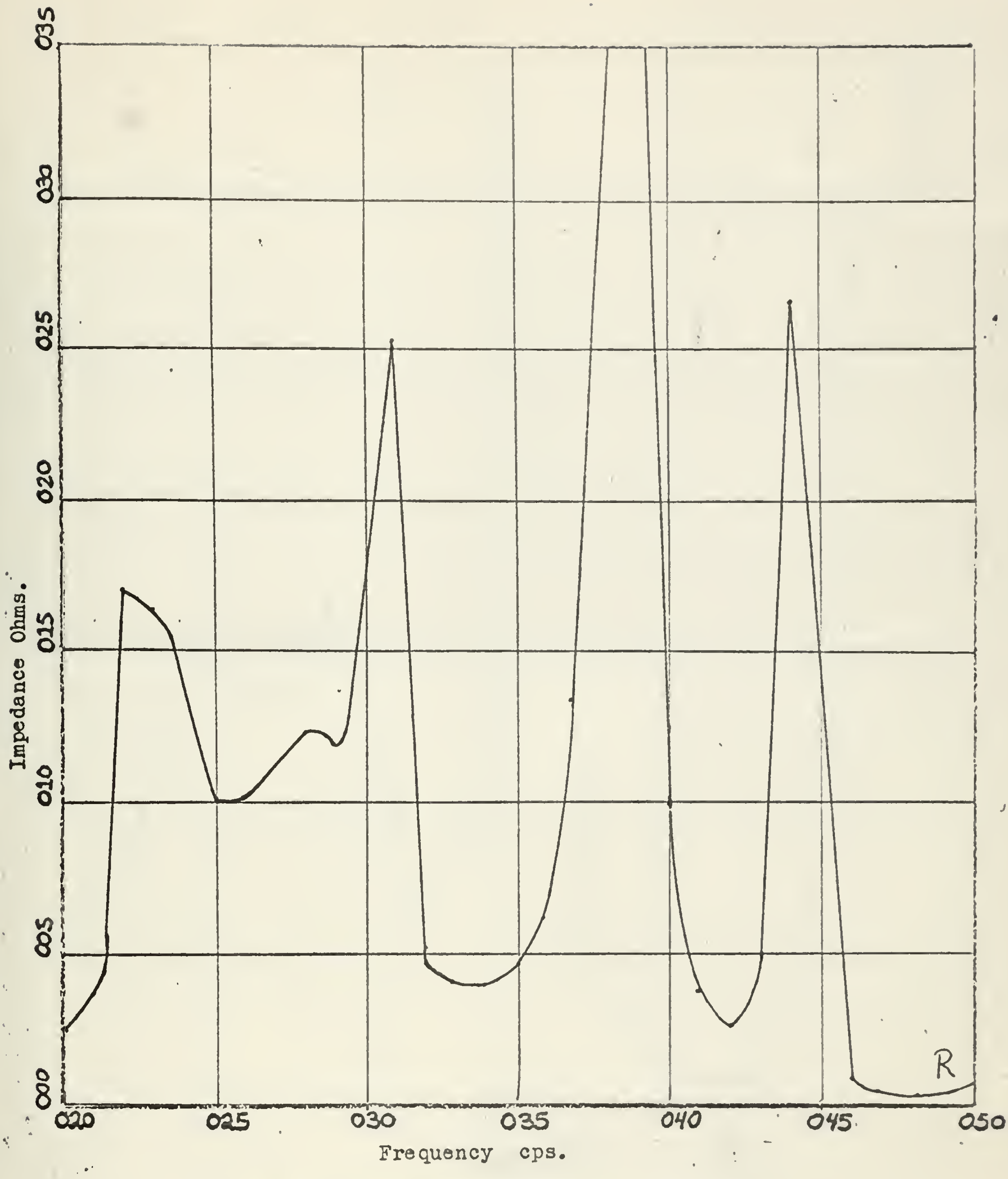


Fig. 4-5. Transducer With Finite Horn.

X-SCALE = 5.00E+02 UNITS/INCH.
Y-SCALE = 5.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY
D=2IN.

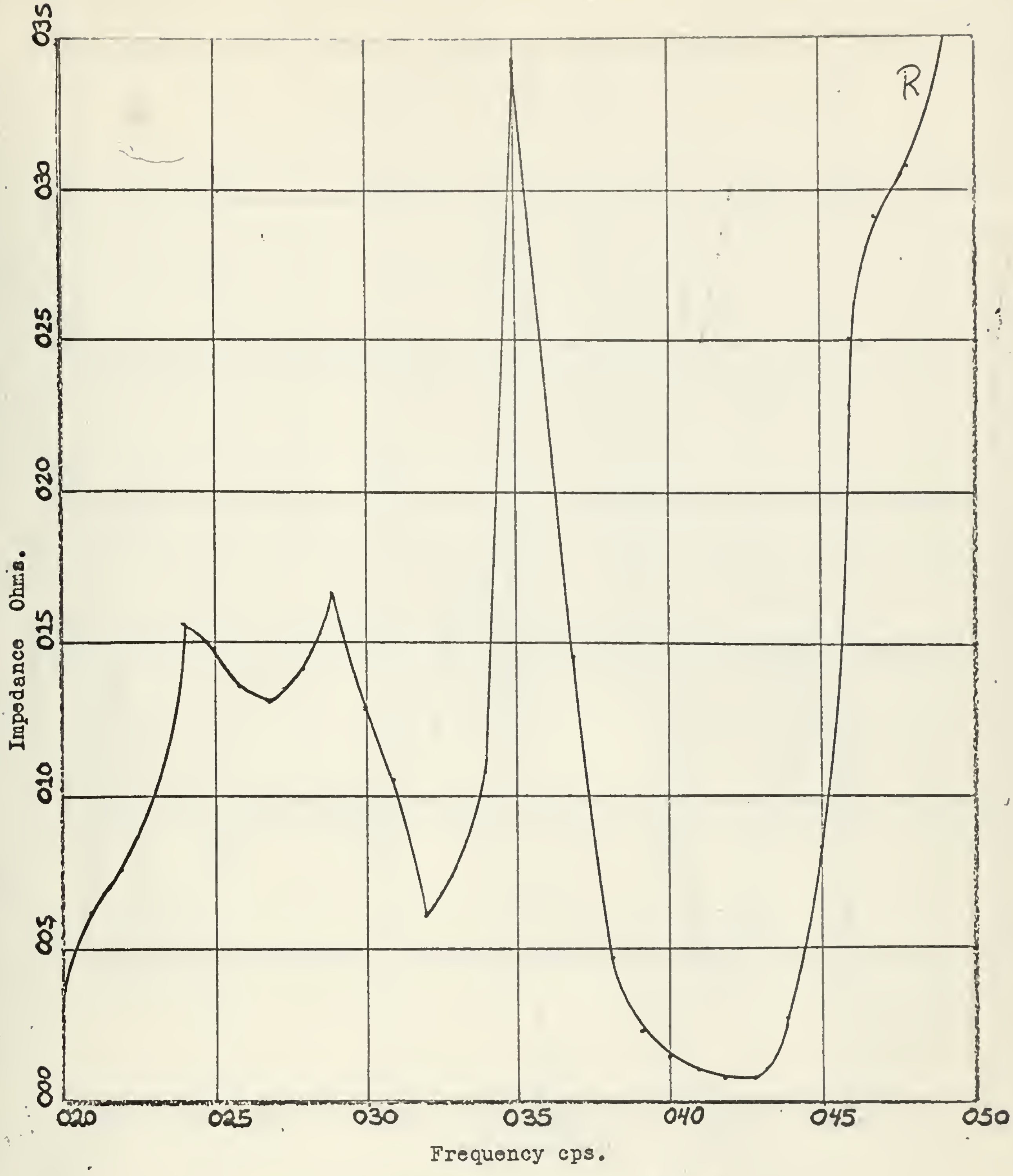


Fig. 4-6. Transducer With Finite Horn.

X-SCALE = 5.00E+02 UNITS/INCH.

Y-SCALE = 5.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY

D=6IN.

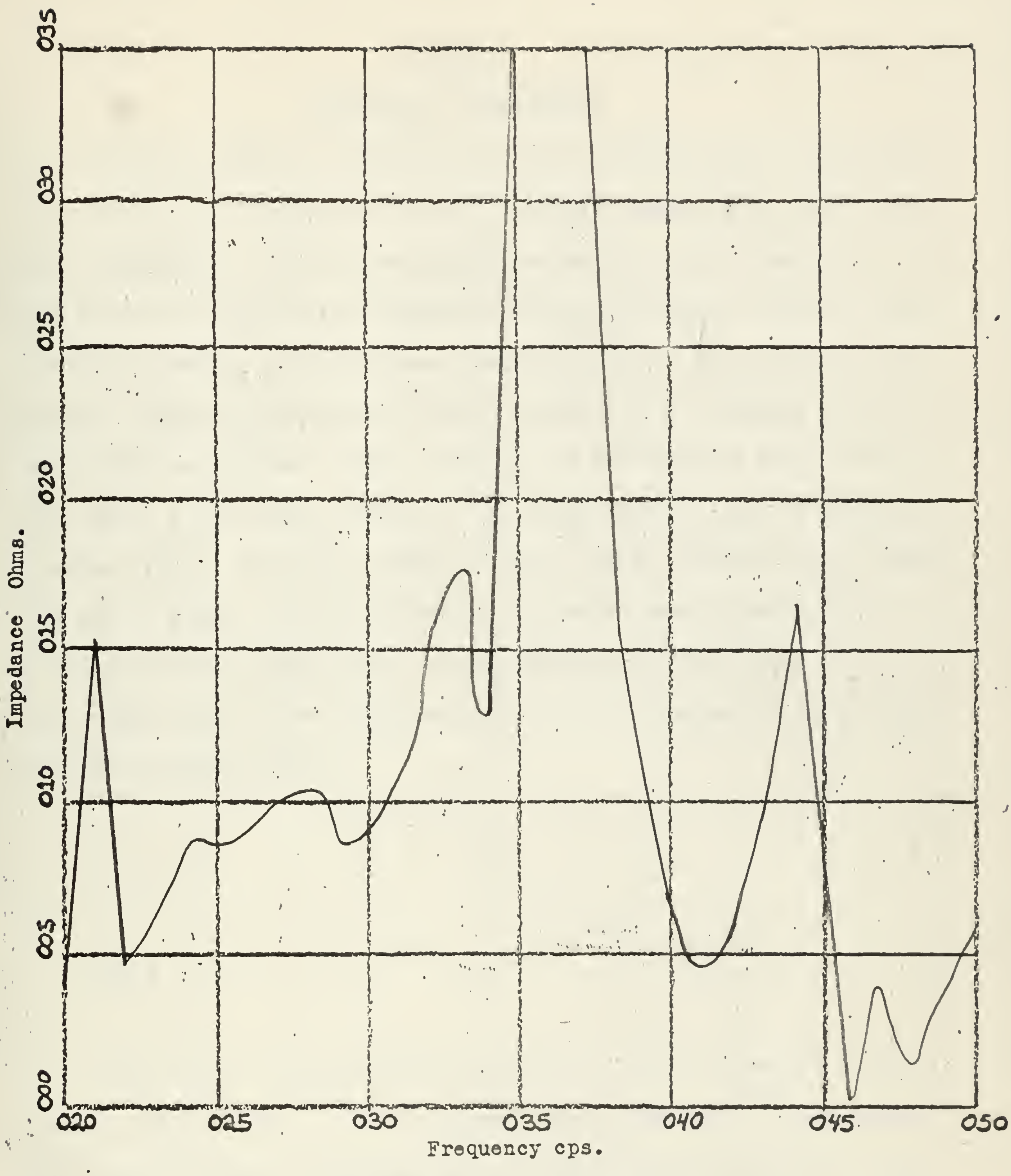


Fig. 4-7. Transducer With Finite Horn.

X-SCALE = 5.00E+02 UNITS/INCH.
Y-SCALE = 5.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY
D=10IN.

CHAPTER V

THEORETICAL CONCLUSIONS

From the curves of the specific acoustic impedance with and without horn developed in Chapters III and IV, several observations can be made. Most important is to note that the reflection due to the interface at the mouth of the horn is an important term and has great bearing on the resistive loading. As can be seen from Figs. 4-4, 4-5, 4-6 and 4-7, at certain frequencies the waves (direct and reflected) reinforce and subtract from one another. Thus, standing wave patterns are setup in the horn that in some cases aid and in some cases detract from the resistive loading effect. The basic reason for this is that the horn is too short for use in water. The high velocity of sound in water versus air dictates the use of longer horns. If, in Equation (4-25), $\overrightarrow{B}$ were set to zero; this would be the condition of an infinite exponential horn. Equation (4-25) then becomes:

(5-1)

Note that in this equation the specific acoustic impedance of the medium within the horn is not dependent on the location of the analysis. The specific acoustic impedance for an infinite horn is plotted in Fig. (5-1).

Also for purposes of comparison, the resistive loading effect is plotted in Fig. 5-2, at a distance 2 inches from the throat of the horn

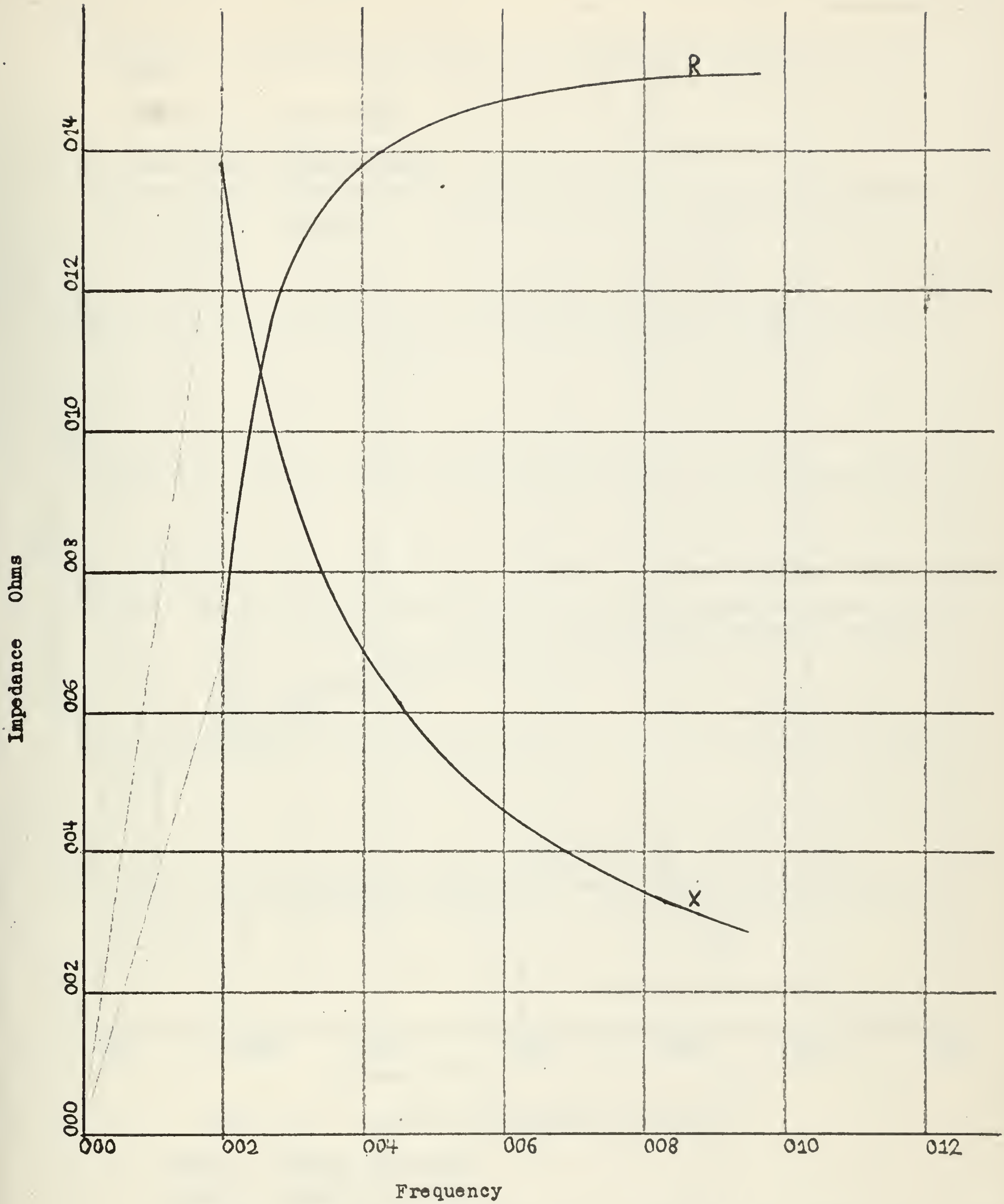


Fig. 5-1. Transducer - Infinite Horn.
X-SCALE = 2.00E+03 UNITS/INCH.
Y-SCALE = 2.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC IMPEDANCE VS FREQUENCY

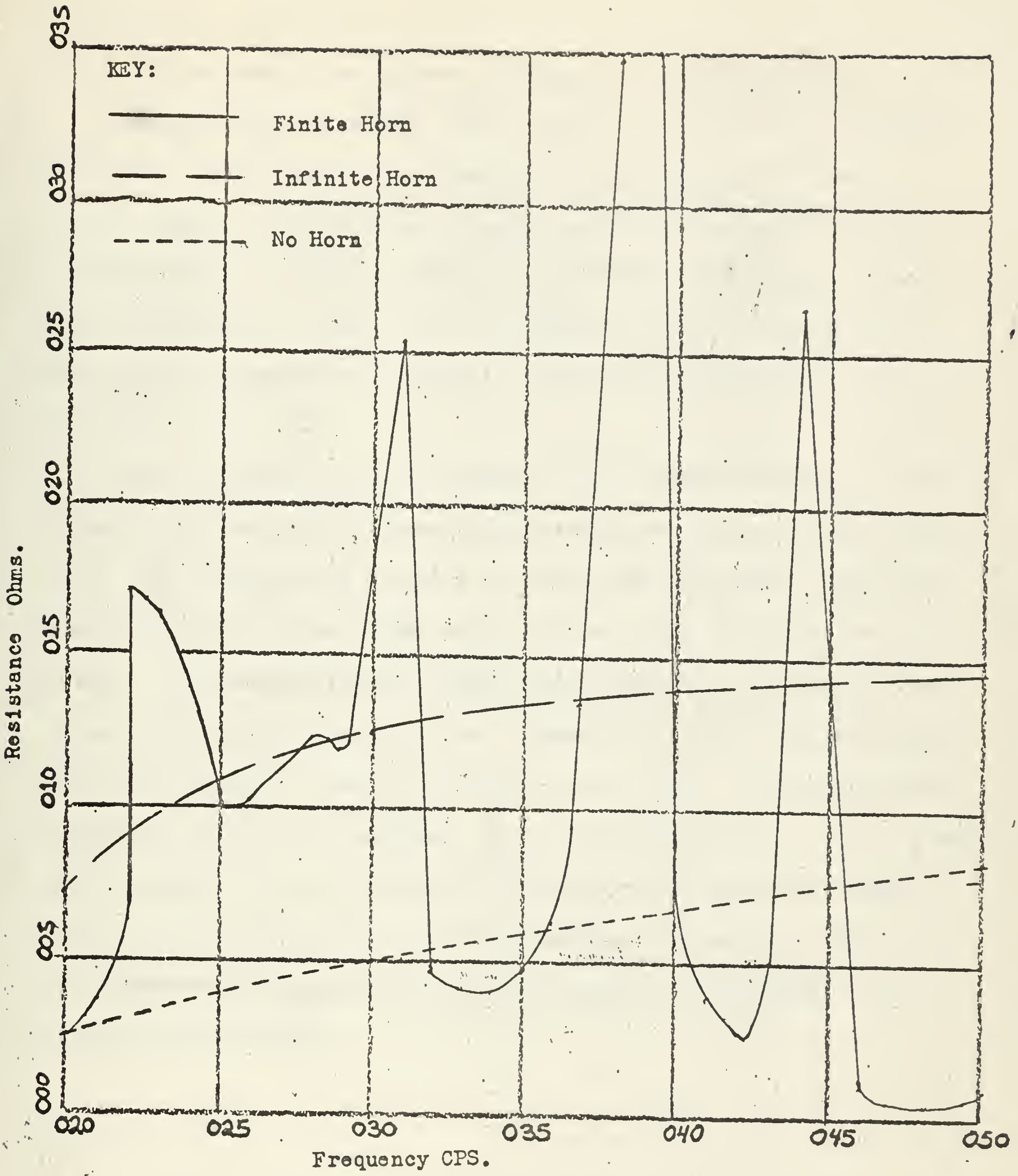


Fig. 5-2. Comparison of Resistive Effects.

X-SCALE = 5.00E+02 UNITS/INCH.
Y-SCALE = 5.00E+04 UNITS/INCH.

SPECIFIC ACOUSTIC RESISTANCE VS FREQUENCY
D=2IN.

for the three conditions: no horn, finite horn and infinite horn.

Another factor concerned in the design of an optimum horn would be the flare of the horn. It was observed from Equation (4-25) that a smaller flare, would result in a higher resistive loading and a lower cutoff frequency. In the case of the horn used in this project, the cutoff frequency was higher than anticipated and this detracted somewhat from the theoretical analysis. The primary concern was to cover a range from 1 to 5 kc.

Also it should be noted, that some of the basic assumptions applied to the finite horn are not exactly applicable and in some cases questionable. The horn was made of steel and not infinitely smooth, therefore, there are small viscous losses where the horn comes in contact with the water. Also partly because of these viscous effects, the waves are not plane waves but have some curvature. These effects are considered small and do not detract a great deal from the actual case. One questionable assumption concerns the stiffness of the walls of the horn. Fig. 5-3 shows that the ρc of steel is comparable to that of sea water and, therefore, the steel horn is to some degree transparent to the medium. There is a loss transversly through the walls of the horn and this reduces the resistive loading effect.

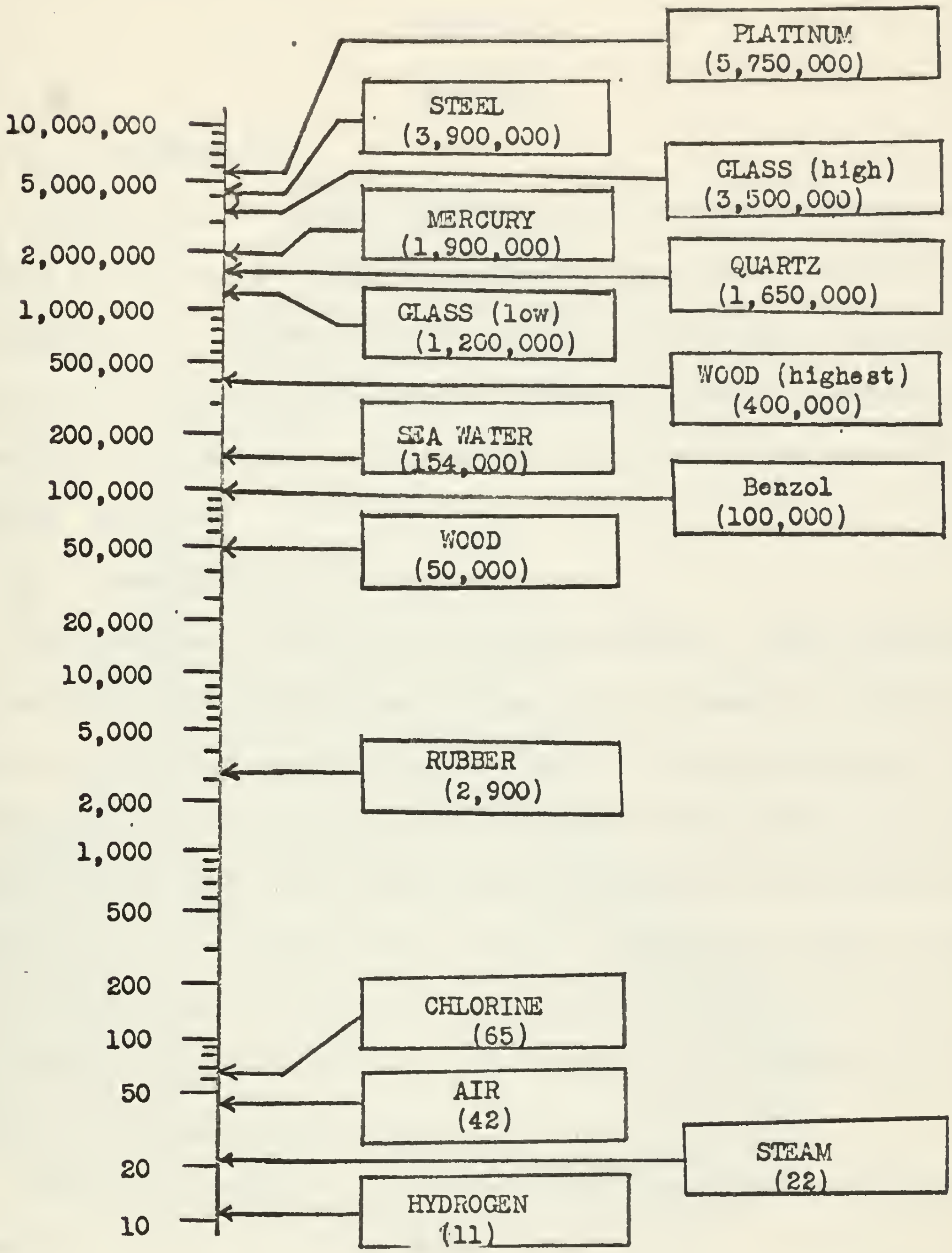


Fig. 5-3. Specific Acoustic Impedance of Various Media.

CHAPTER VI

EXPERIMENT

A. Introduction.

The main purpose of the experiments performed was to analyze the effects of coupling a horn to an underwater transducer. This was done by comparing the transducer with horn versus the transducer without horn as to admittance and impedance data, frequency response, and beam pattern. These areas will be taken up individually and the results shown in the ensuing sections.

B. Description of the Transducer.

The transducer used was a low frequency device. The electro-mechanical conversion was accomplished through ceramic elements. Since the transducer is classified, only this general description can be given. This restriction, however, does not detract from the understanding and development of the analysis. The transducer was modified by enclosing it between two metal rings fitted with spacers. This was necessary to prevent the transducer from being used in a clamped mode when coupled to the horn. The transducer is shown coupled to the hornin Fig. 6-1 and Fig. 6-2 .

C. Description of the Horn.

The horn is known as a catenoidal horn. It was designed with a curvature conforming to the equation:

$$S_{\ell} = S_o \cosh^2\left(\frac{x}{q}\right)$$

where: S_{ℓ} = cross-sectional area of the horn

S_o = cross-sectional area at the throat of the horn

The dimensions in Table I are explained in Fig. 6-3.

Fig. 6-1

View of Horn Coupled to Transducer

Fig. 6-2

View of Horn Coupled to Transducer

The horn was fabricated out of steel stock using the following design data:

TABLE I: $S_{\ell} = S_o \cosh^2\left(\frac{x}{h}\right)$

$d_{\ell} = d_o \cosh\left(\frac{x}{h}\right)$

mouth area = 1645 cm

h = flare constant = 9"

X inches	X/9 inches	$\cosh\left(\frac{X}{9}\right)$	dx inches	ϕ = slope	A sq. in.
0	0	1.000	4,000	0° 0'	12.566
1	.1111	1.0062	4,025	1° 25'	12.724
2	.2222	1.0248	4.099	2° 52'	13.198
3	.3333	1.0561	4,224	4° 20'	14.013
4	.4444	1.1002	4.401	5° 51'	15.212
5	.5556	1.1583	4.633	7° 25'	16.859
6	.6667	1.2306	4.922	9° 6'	19.027
7	.7778	1.3180	5.272	10° 49'	21.829
8	.8889	1.4215	5.687	12° 41'	25.401
9	1.0	1.5431	6.172	14° 50'	29.919
10	1.111	1.6834	6.734	16° 50'	35.616
11	1.222	1,8456	7.382	19° 10'	42.800
12	1.333	2.0287	8.115	21° 30'	51.721
13	1.444	2.2376	8.950	24° 2'	62.913
14	1.556	2.4742	9.897	26° 42'	76.931
15	1.667	2.7417	10.967	29° 38'	94.464
16	1.778	3.0423	12.169	32° 38'	116.306
17	1.889	3.3814	13.526	35° 46'	143.691
18	2.000	3.7622	15.049	39° 0'	177.871
19	2.111	4.1892	16.757	42° 18'	220.538
19.66	2.1846	4.5000	18.000	44° 22'	254.470

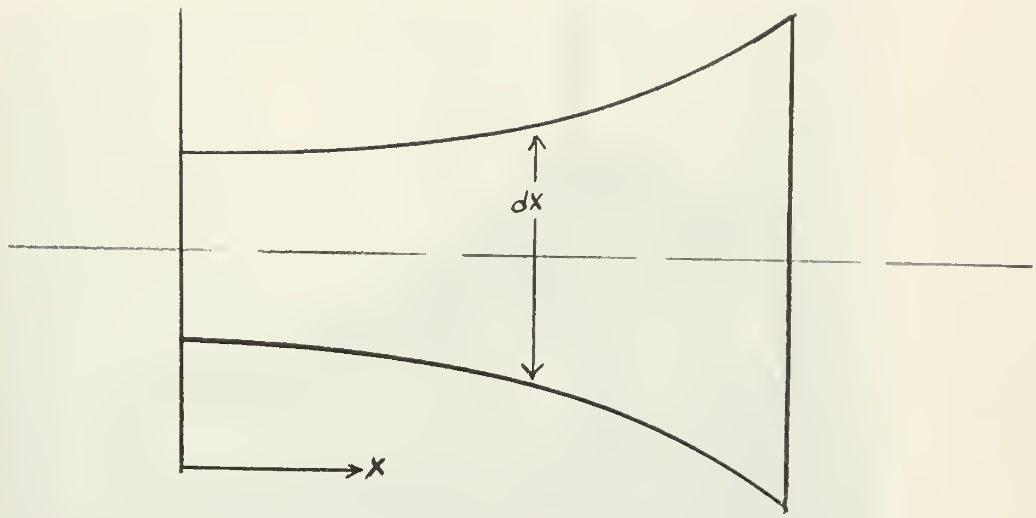


Fig. 6-3

Dimensional View of the Horn

Fig. 6-4 and Fig. 6-5 show the horn in various stages of fabrication. Fig. 6-1 and Fig. 6-2 show views of the horn coupled to the transducer. The horn is 1/2 inch thick and weights approximately 90 pounds.

D. Admittance Bridge.

The data taken on the admittance bridge is considered the most important because it shows the phase relationship in impedance. The impedance taken on the bridge is the equivalent impedance seen looking into the terminals of the transducer. Therefore, not only the impedance of the medium is present but also the electrical and mechanical impedance of the transducer. This overall impedance is necessarily different than considering the impedance of the medium by itself. The purpose of the experiment is a comparison of the resistive and reactive loading effects for the transducer with and without horn. Since all other parameters remain the same except the impedance effects on the piston in both instances, the

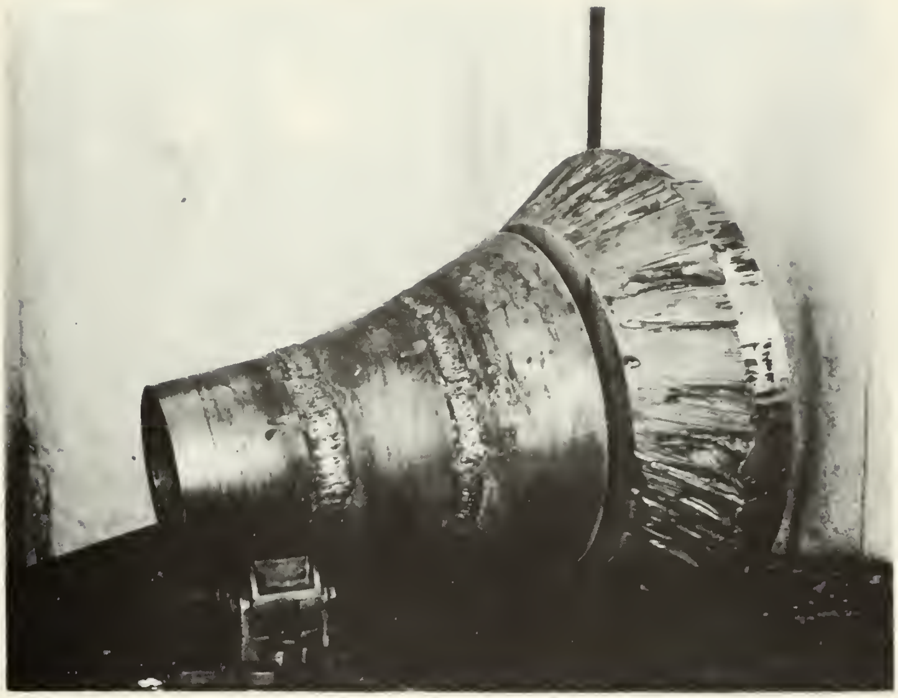


Fig. 6-4

Fabrication Views of Horn

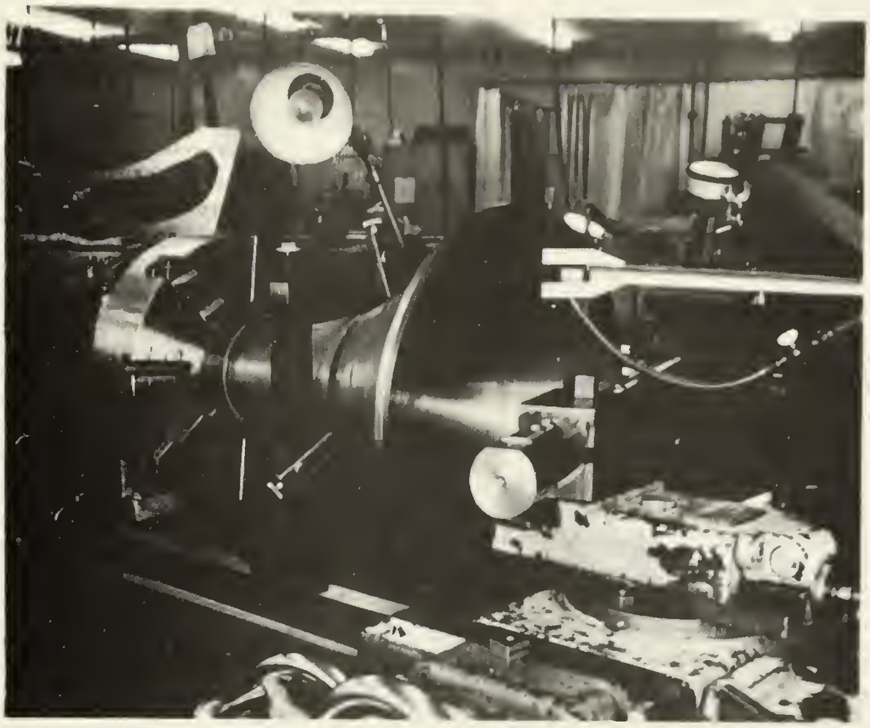
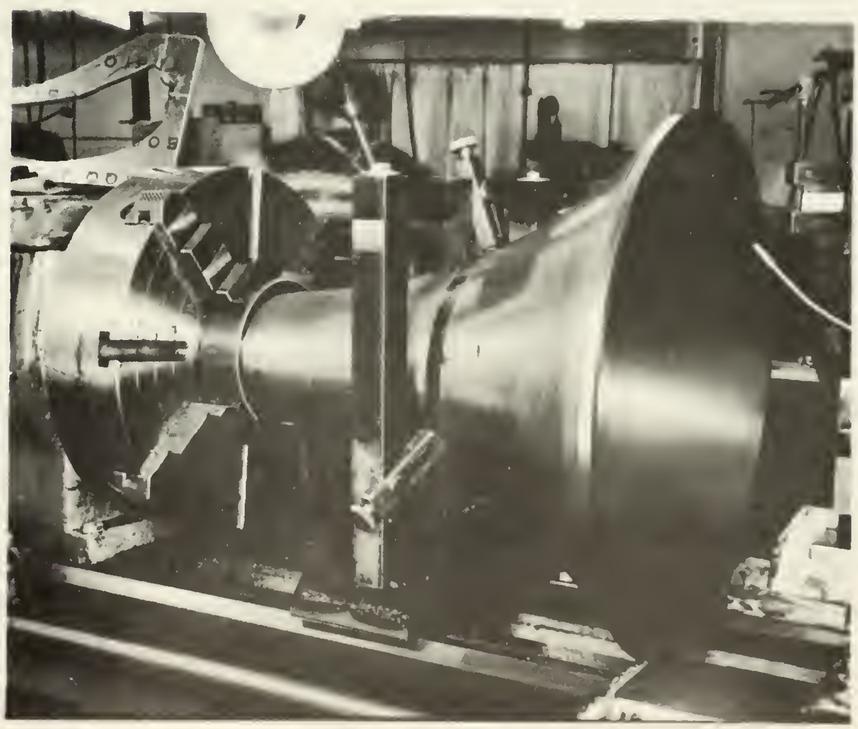


Fig. 6-5

Fabrication Views of Horn



gain or loss in resistive loading can be effectively compared between experimental results and the theory without separating the various impedances taken in the experiment.

A block diagram of the experimental setup is shown in Fig. 6-6. A variable frequency sine generator was used as the signal source. The bridge was nulled at preset frequencies using an oscilloscope. A band pass filter was used in conjunction with the scope to eliminate unnecessary noise and allow greater accuracy in nulling the scope. It should be pointed out that great care had to be taken in obtaining data around resonant points. In some instances the resonance peaks occurred with a width of less than one hundred cycles at a frequency of two to ten thousand cycles per second. The transducer was immersed in a salt water tank and aimed at the far corners. Tank dimensions are given in Fig. 6-7 and Fig. 6-8. The transducer was suspended on a cable and allowed to rotate slowly about the far corner of the tank. The reason for aiming at the far corner and the slow rotation was an attempt to break up any standing wave patterns that may develope and therefore decrease radiation back to the transducer. The signal was restricted to low power for the same reason. Because of the limitations of the tank and the low frequencies used, there was some radiation back to the transducer but these effects are neglected in the analysis. In each instance, before the transducer was immersed in the water; it was washed with a suitable wetting agent to eliminate air bubbles as this would reduce the loading effects.

The admittance data taken was converted to impedance data and is shown on Fig. 6-9 and Fig. 6-10. The resistive effects are compared on Fig. 6-11. Fig. 6-9 and Fig. 6-10 show the difference in resistive loading and reactive loading. In both cases the reactive impedance was much

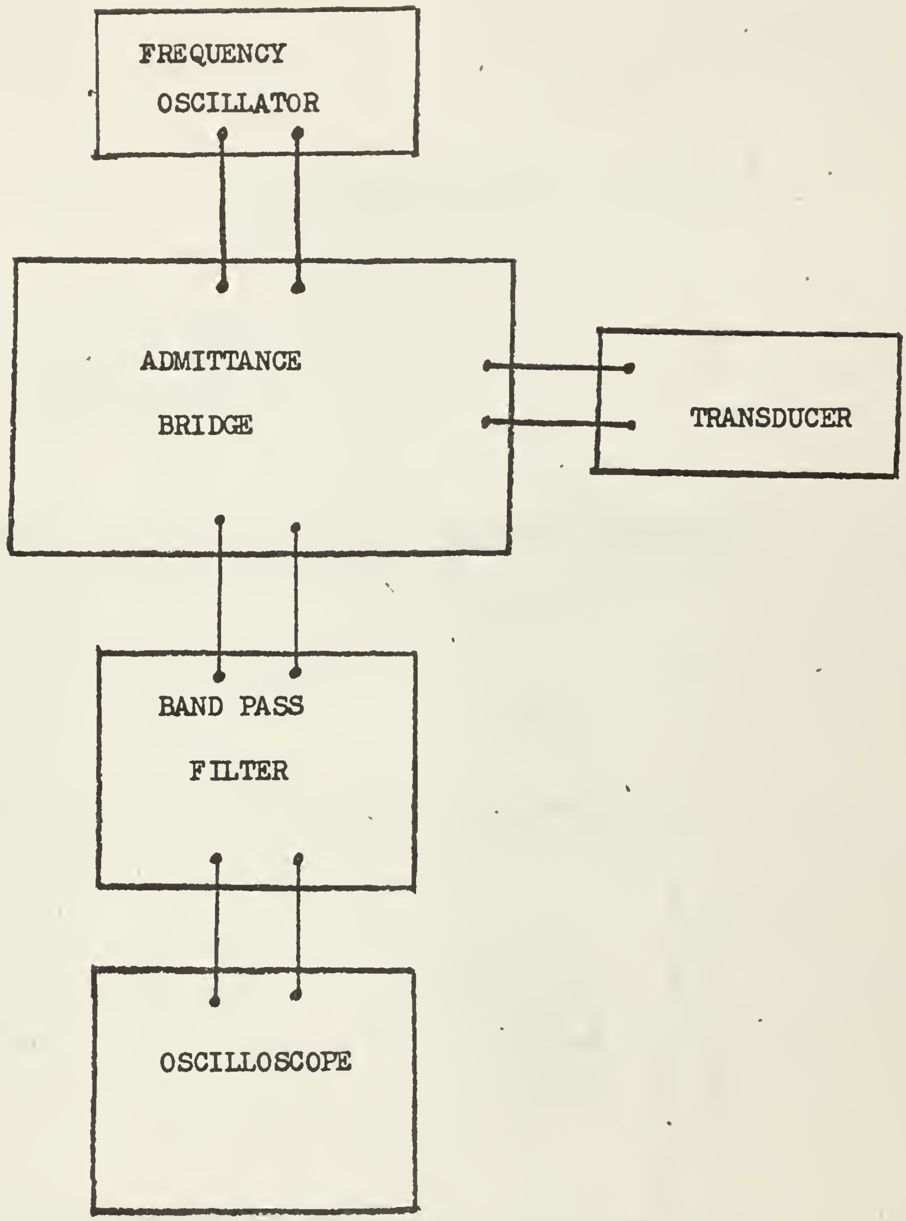


Fig. 6-6. Block Diagram of the Admittance Bridge.

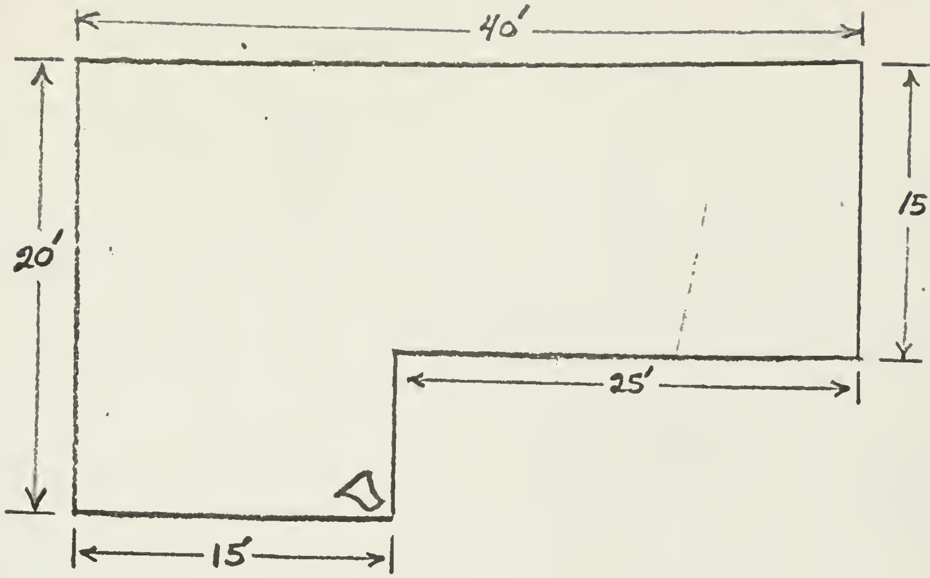


Fig. 6-7. Top View of Test Tank with Transducer

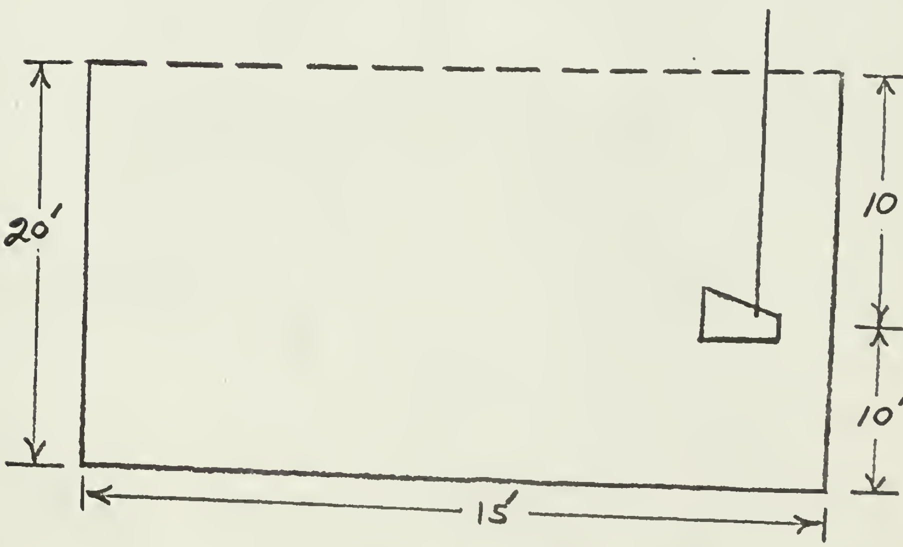


Fig. 6-8. Side View of Test Tank with Transducer

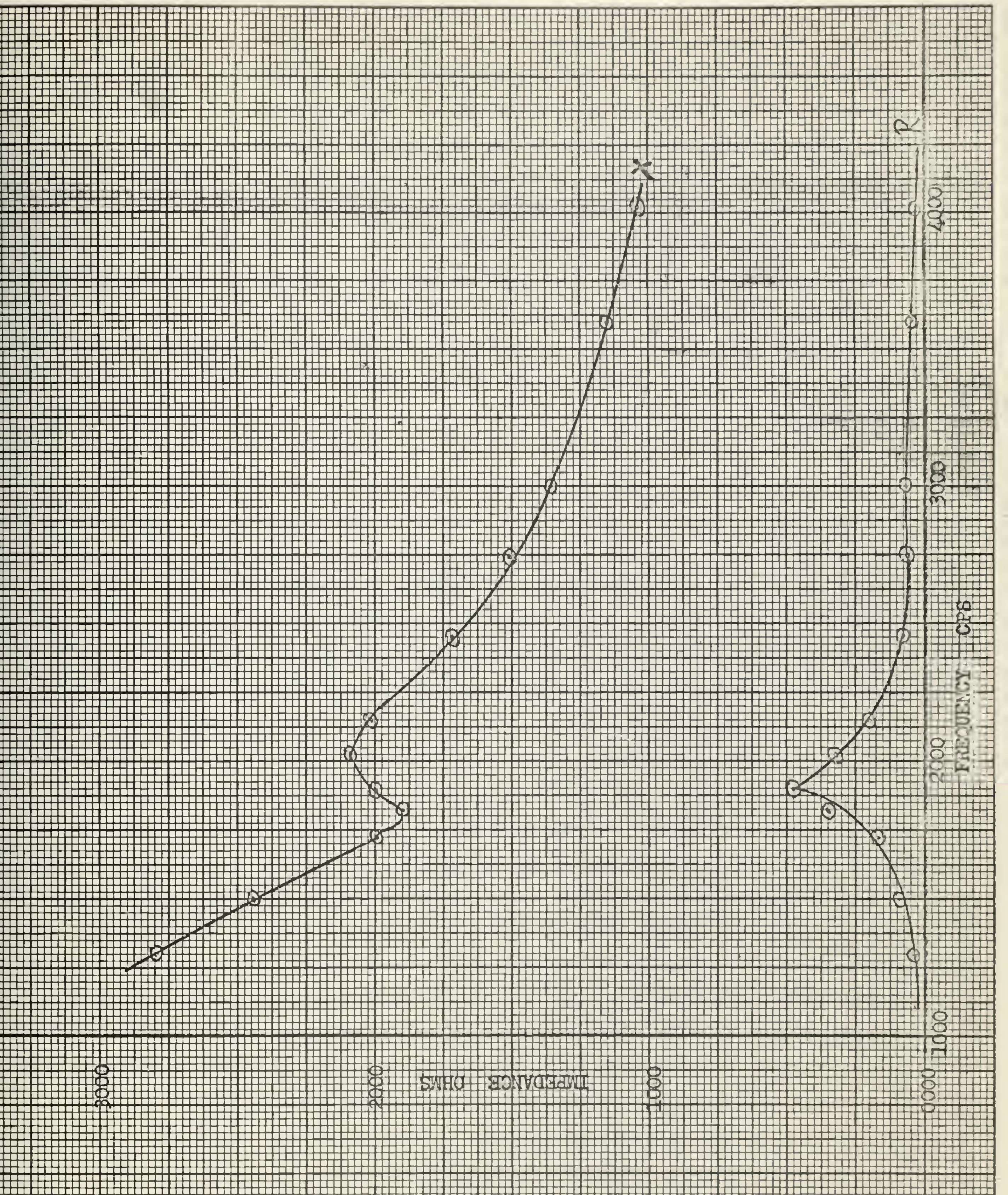


Fig. 6-9 Impedance Data for No Horn in Water

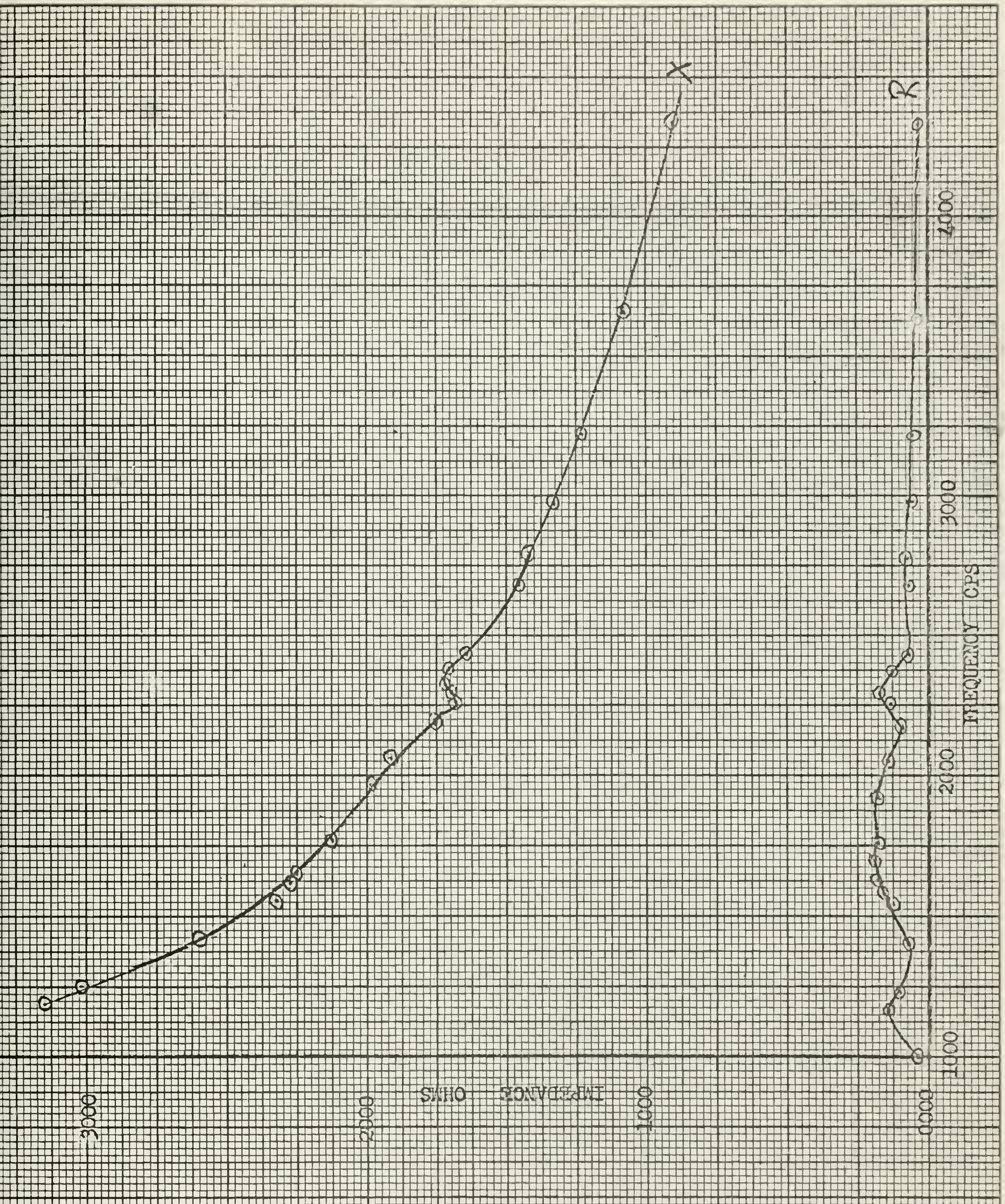


Fig. 6-10 Impedance Data for Horn in Water

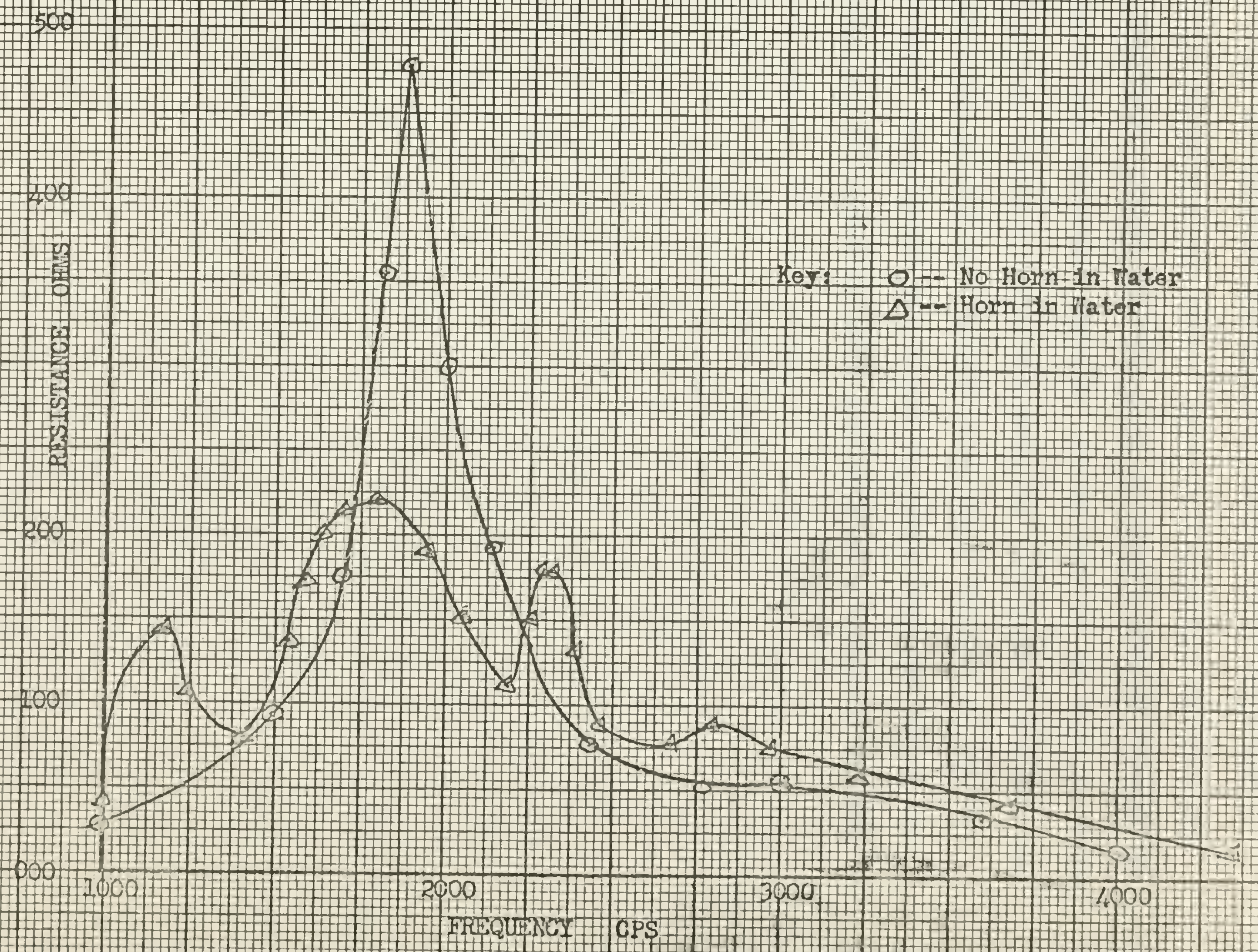


Fig. 6-11 Comparison of Resistive Effects

greater than the resistive effects. These figures also show that the lower frequencies are the most inefficient. In Fig. 6-11, the case of no horn in water shows one very peaked resonant frequency. At this frequency, the resistive effects are very good but over a band of frequencies, the resistive effects attenuate rapidly. In the case of the horn in water, it is observed that the horn definitely made the transducer a broader band device but in doing so drastically attenuated the resistance at the resonant frequency. Also notice that there is a gain with the use of the horn below the cut off frequency and a good gain at a frequency of 1200 cps.

E. Vector Impedance Locus Plot Data.

Impedance data was taken on the Vector Impedance Locus Plot (Abbreviated V.I.L.P.) as a back up to the data taken on the Admittance Bridge. The block diagram for this experimental setup is shown in Fig. 6-12. The experimental technique involved is similar to that for the Admittance Bridge. The data taken on the V.I.L.P. is considered accurate to only ten percent whereas the admittance data is considered accurate to one percent. For this reason, the V.I.L.P. data was taken only to corroborate the Admittance data and therefore no analysis is made of this data. The experimental curves are shown in Figs. 6-13 and 6-14.

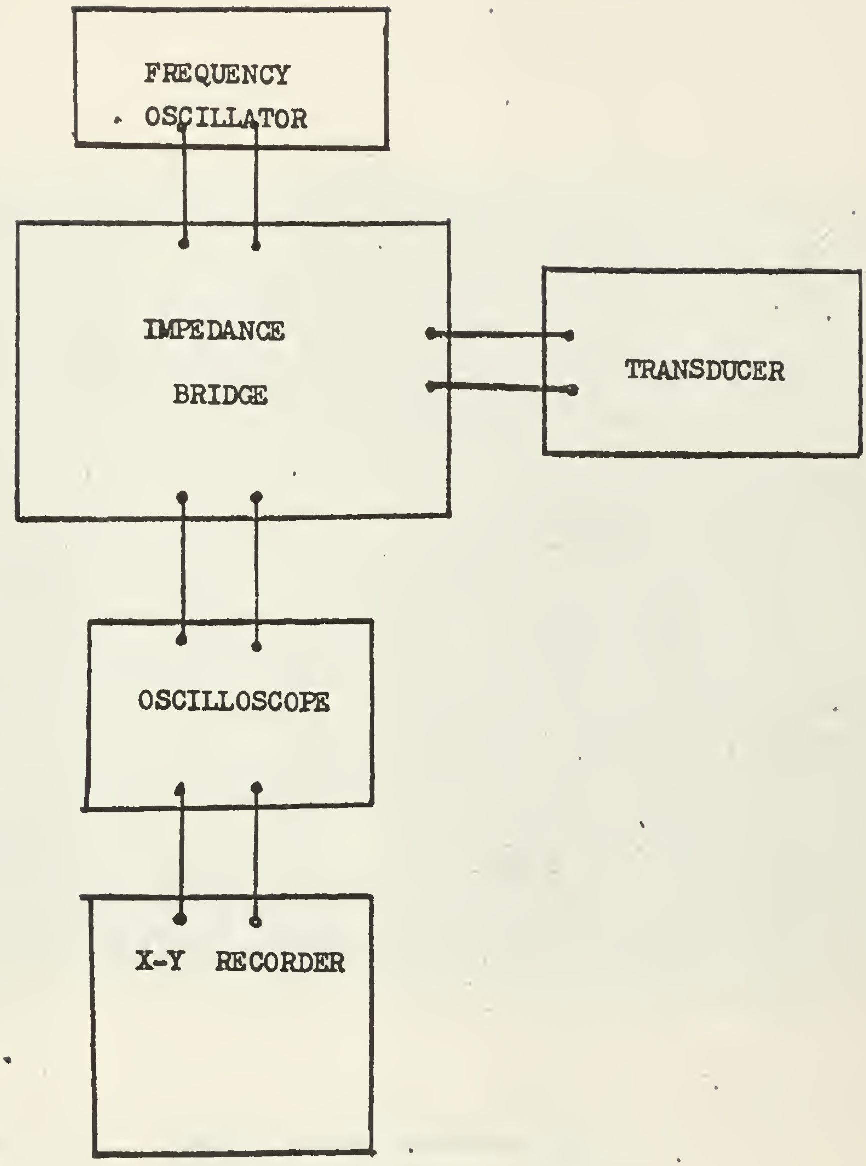


Fig. 6-12. Block Diagram of the Impedance Bridge.

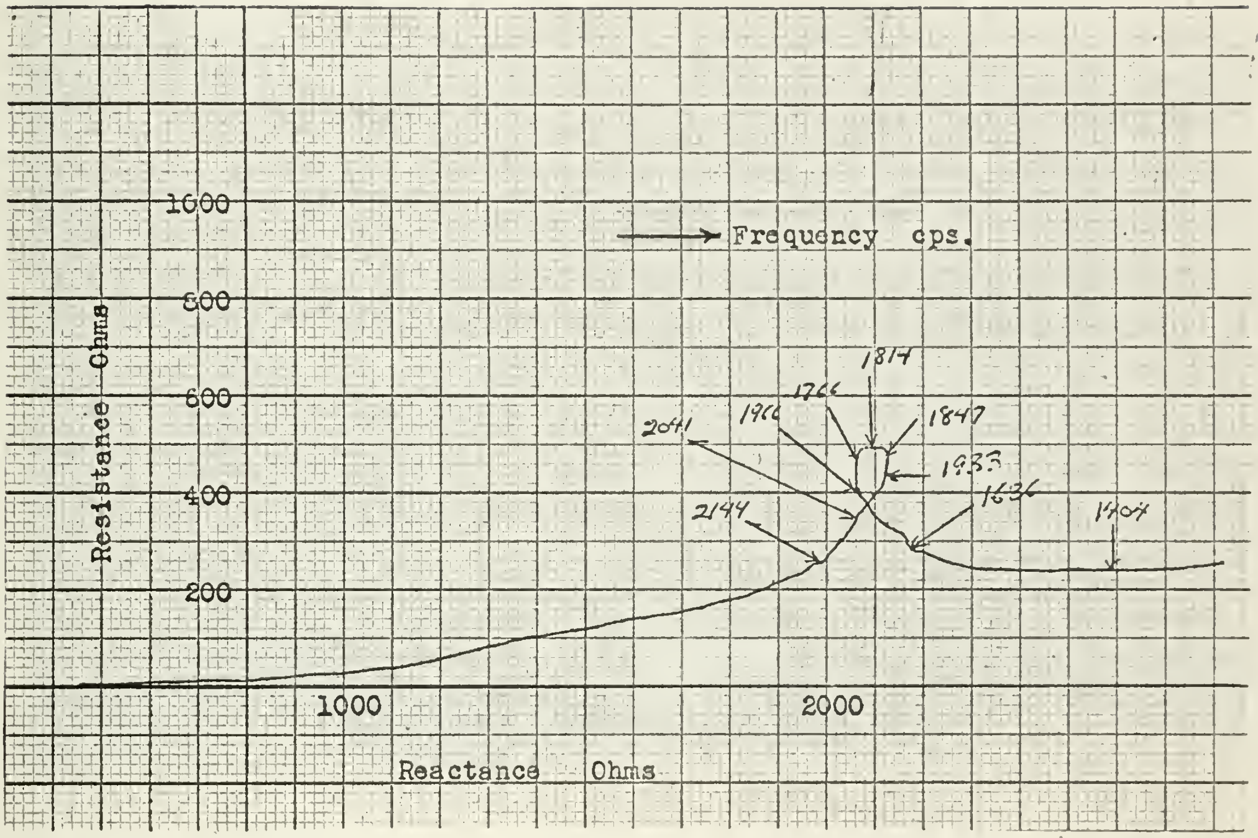


Fig. 6-13. Vector Locus Impedance Plot
Transducer in Water - No Horn.

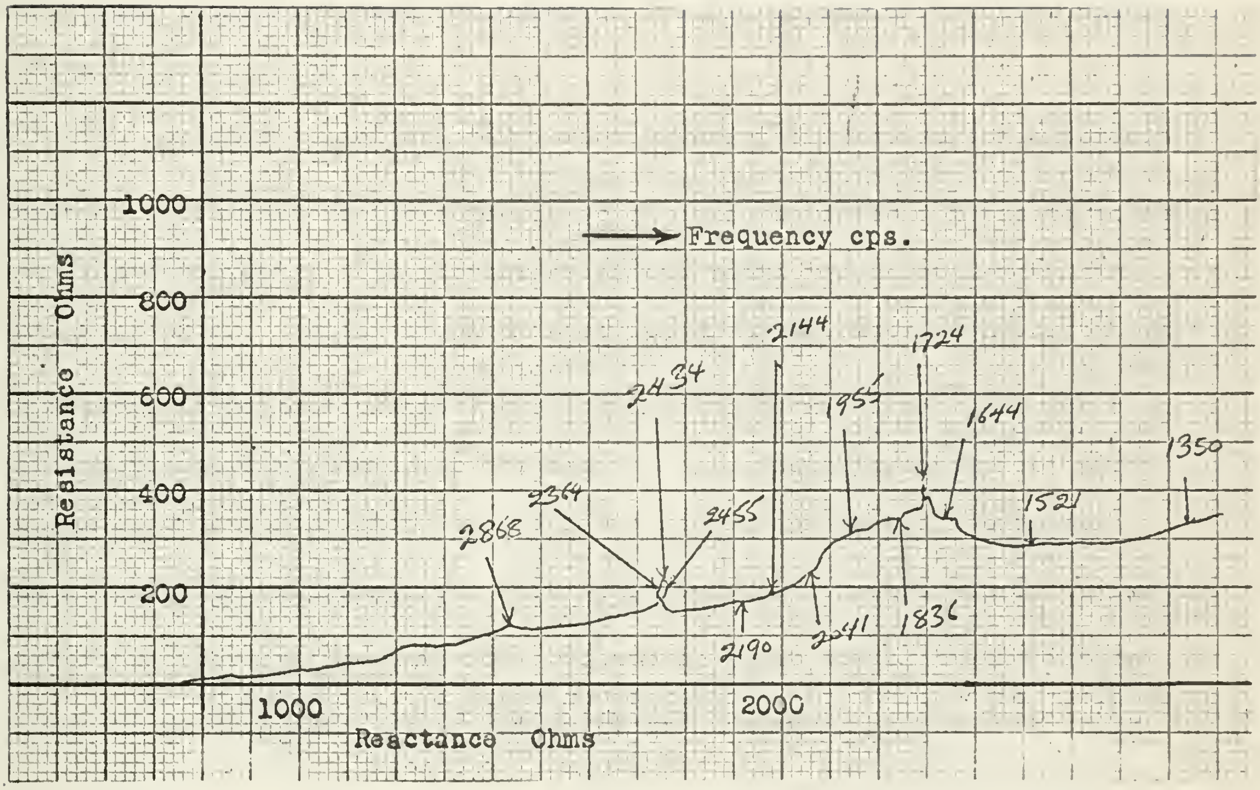


Fig. 6-14. Vector Locus Impedance Plot
Transducer in Water - With Horn.

F. Frequency Response.

In order to obtain frequency response data, the transducer was connected to a variable frequency oscillator and allowed to transmit in water. The front face of the transducer was directed to a calibrated hydrophone which monitored the output. The block diagram for this system is shown in Fig. 6-15.

The Oscillator and Send Modulator combination delivers a sinusoidal signal of low level to the Transmit Gate. Gating pulses are received from the Pulse Generator. The parameters of the pulse can be varied by the Pulse Generator. (Pulse width .1 millisecond to 60 millisecond and pulse repetition rate 4 milliseconds to 1 second). The signal is then fed to the Power Amplifier and on to the transducer. The power output of the transmitter is monitored in db. on a Ballantine Vacuum Tube Voltmeter. The pulse width and repetition rate transmitted by the transducer is very critical at low frequencies. The pulse must reach the calibrated hydrophone and be completed before radiation off the walls of the tank reach the hydrophone. Also, because of reactive components in the transmit section, the first two or three cycles of the sinusoidal pulse are transient and enough cycles must be allowed in the pulse to reach steady state and transmit a steady state pulse. This transient effect can be monitored on the oscilloscope and thus controlled. However, at low frequencies the pulse length increases beyond the necessary cutoff due to radiation off the walls and this is the limit of the Raytrac system. Since both the transmit pulse and receive pulse can be observed on the oscilloscope, it is approximately known when this limit is reached. Any frequency below 1000 c.p.s. cannot be recorded.

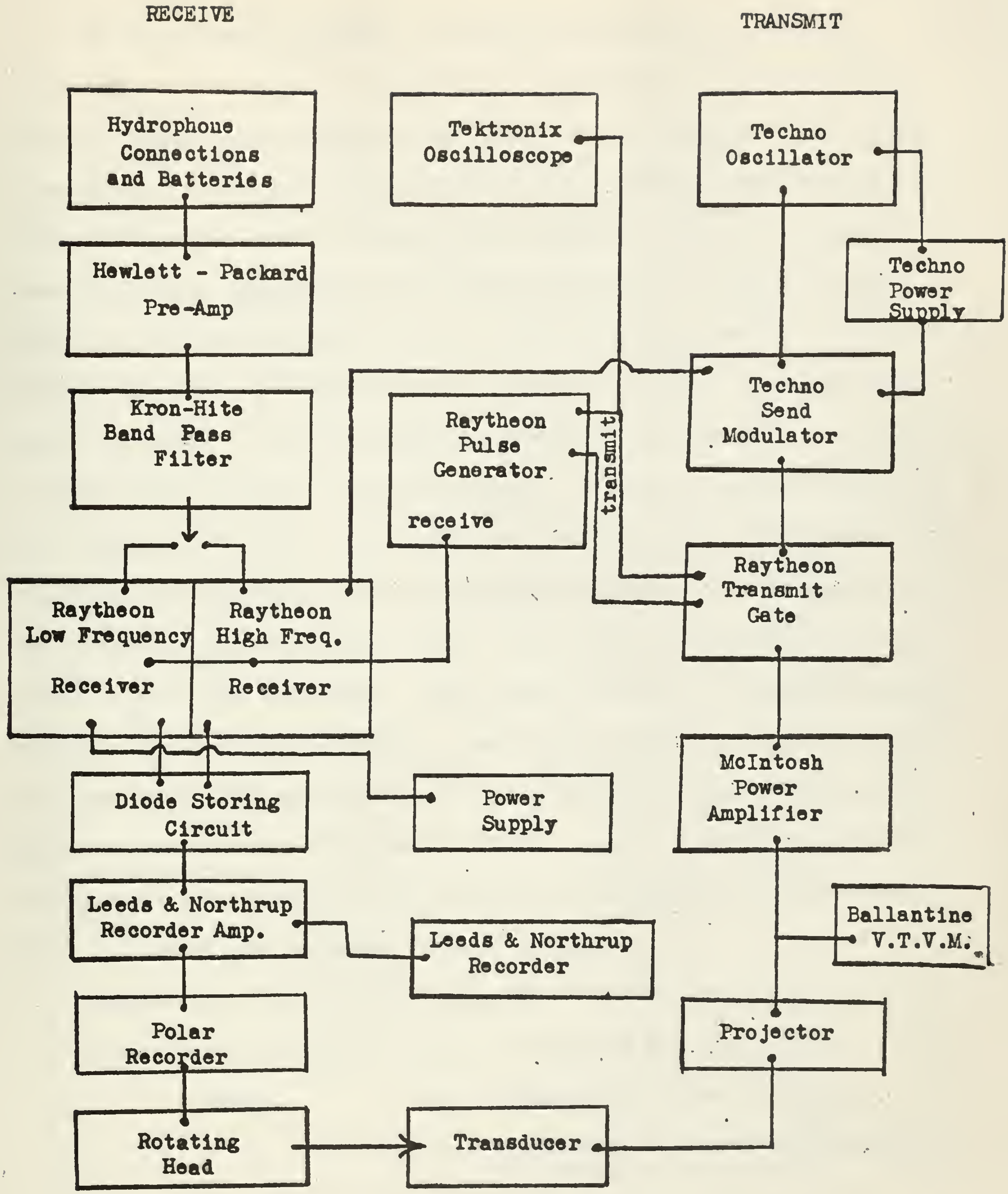


Fig. 6-15. Block Diagram of Raytrac System.

The calibrated hydrophone is energized by batteries external to the circuit. The signal is picked up by the hydrophone and fed to the Pre Amp. where gain selection of 20 and 40 db. are available. The signal then goes to a band pass filter which is set at maximum noise rejection. The ensuing signal goes to either a Low Frequency Receiver or High Frequency Receiver depending on the transmit frequency. The Low Frequency Receiver is a gated amplifier and the output signal to sent to a diode storage circuit. The High Frequency Receiver is similar to the Low Frequency Receiver. The diode capacitor storage circuit makes the receiver a peak reading device. The Pulse Generator is used to gate the High and Low Frequency Receivers. The peak signal is then fed to the Recorder Amplifier and Recorder. After running the experiment, the hydrophone is calibrated on the same curve. Fig. 6-16 and Fig. 6-17 shows the experimental curves that were taken. Using these curves, the frequency response (power output) can be calculated. Before calculating the power output it is necessary to determine the directivity index which can be calculated from the beam patterns (See Section VI-G). Fig. 6-18 shows the physical location of the transducer and calibrated hydrophone for the experiment. Fig. 6-19 shows the frequency response curve.

The equation used in calculating the frequency response curve is:

(6-1) $P \text{ output} = V_H - \Delta - V_c + 20 \log d - E - 71.6 - DI$

where:

V_H = Hydrophone calibration in db.

Δ = Difference in experimental curves in db. (See Figs. 6-16, 6-17).

V_c = TP-210 #15 Hydrophone constant in db.

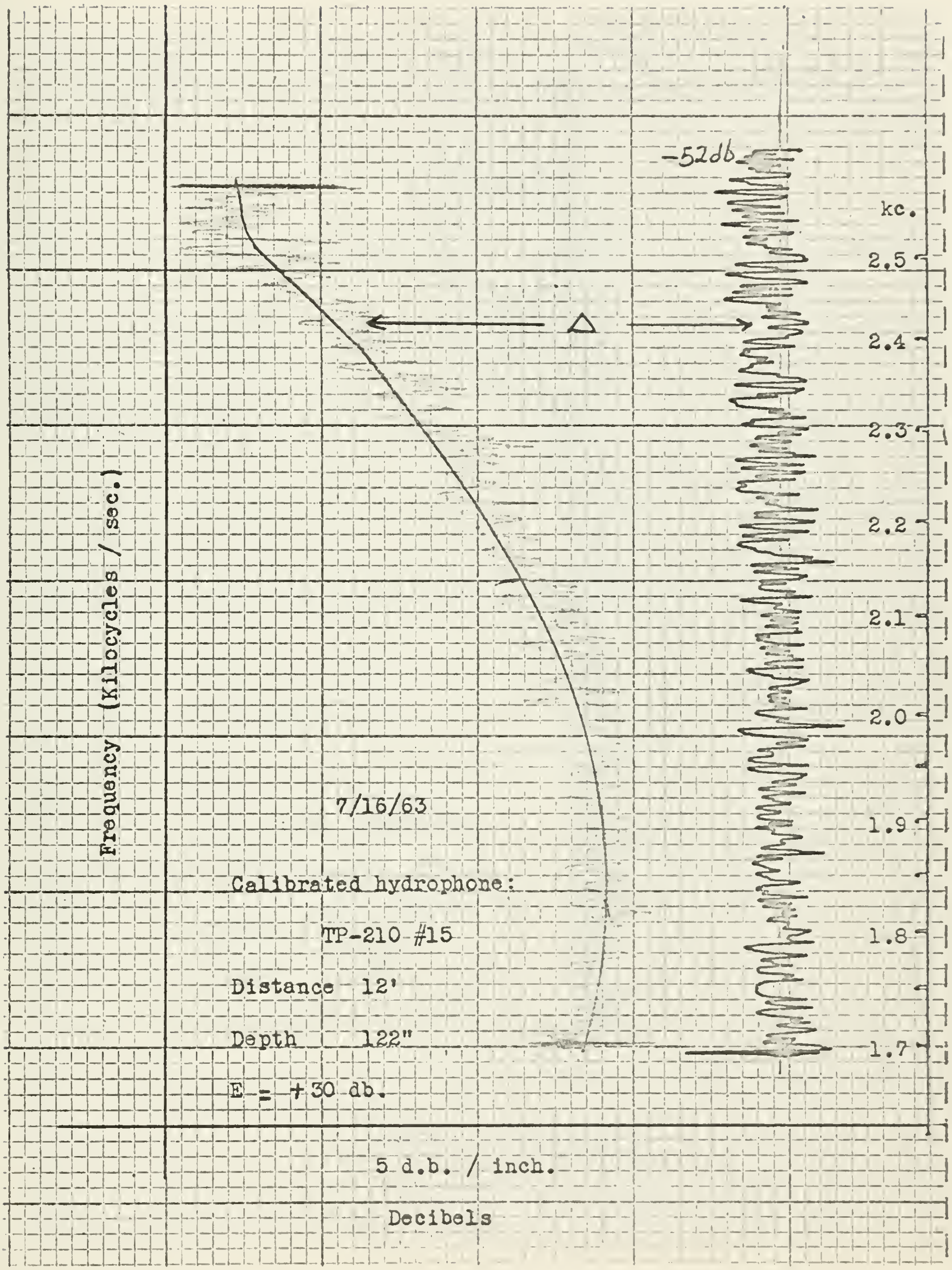


Fig. 6-16. Experimental Frequency Response Curve for Transducer without Horn.

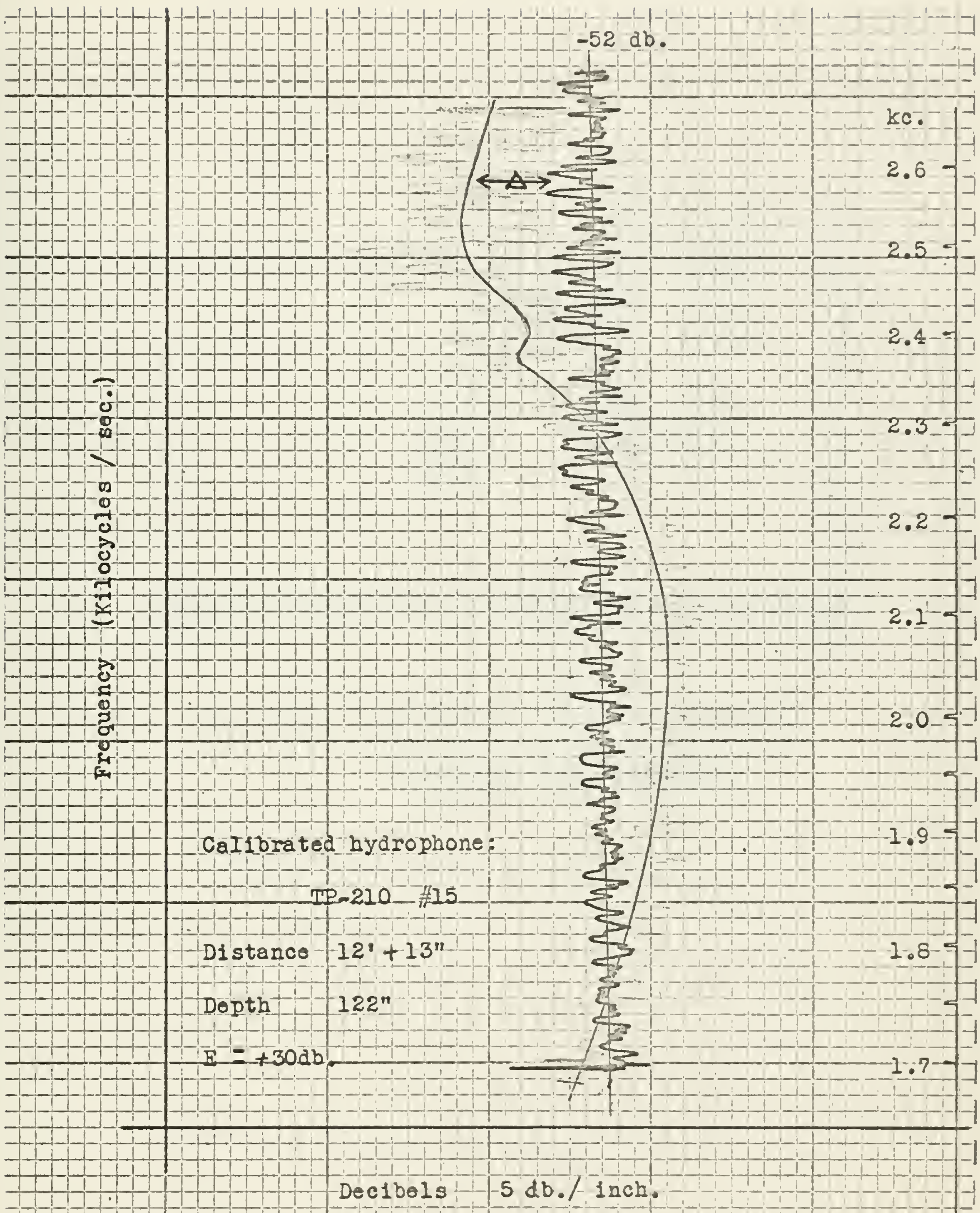


Fig. 6-17. Experimental Frequency Curve
for Transducer with Horn.

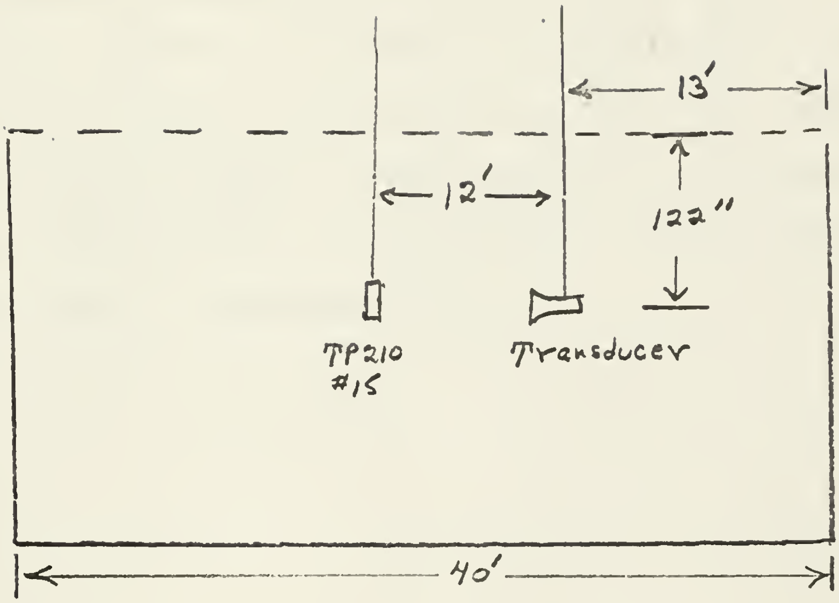


Fig. 6-18. Side View of Tank with Calibrated Hydrophone and Transducer.

d = distance between transducer and hydrophone in yards.

E = Power into transducer in db.

DI = directivity index in db.

The frequency response equation includes the term "DI" which adds to the power output. The more directive the beam the larger in db. this term will become. This helps to account for the greater gain in decibels shows on Fig. 6-19 than is shown by the admittance data. Because of the low frequencies used, there is some question as to the accuracy of these frequency response curves as mentioned above. The curve (Fig. 6-19) shows an overall gain of approximately 10 db.

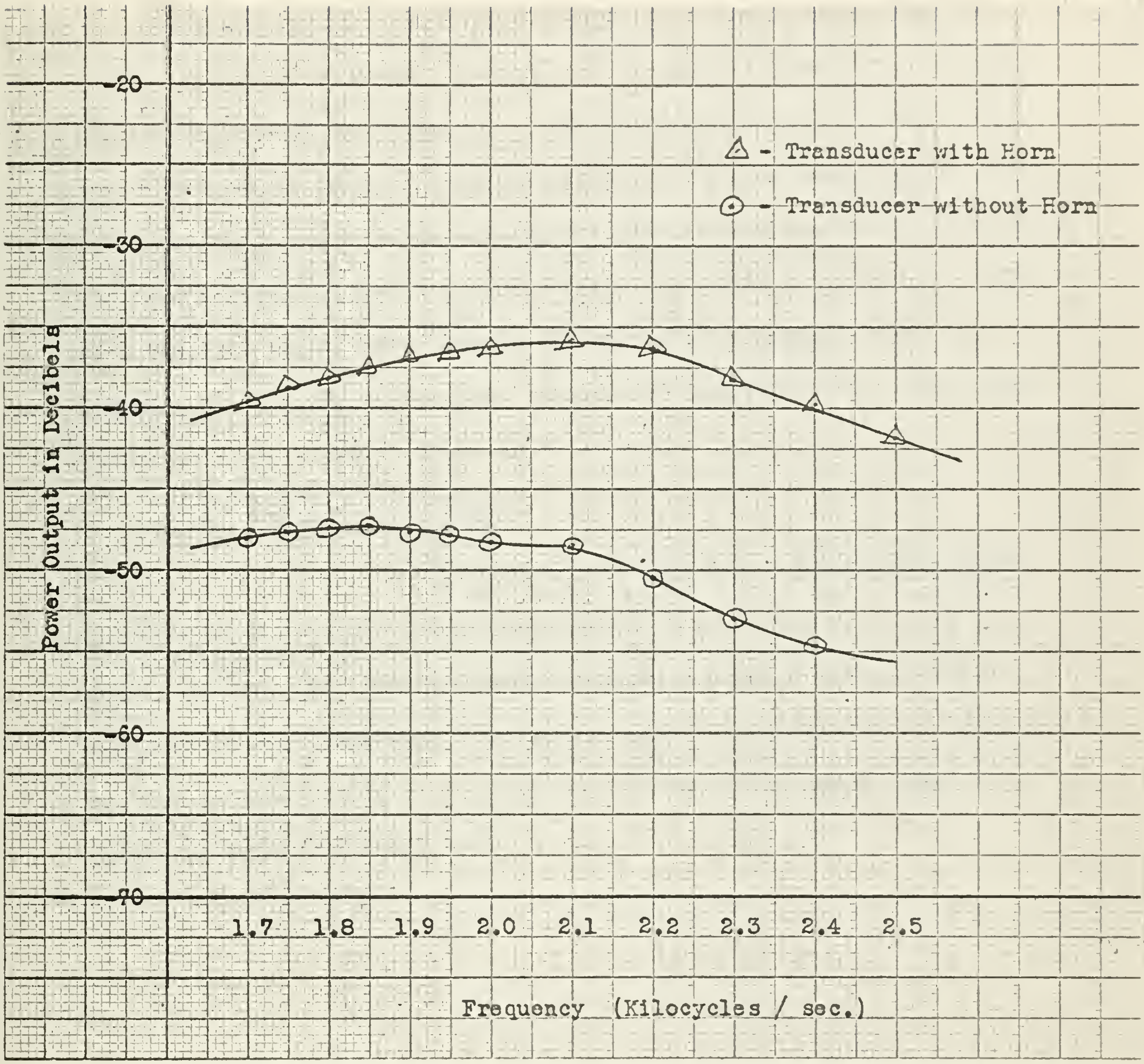


Fig. 6-19. Frequency Response Curves

G. Beam Patterns

In obtaining beam patterns, the Raytrac system is used (See Fig. 6-15). The Polar Recorder is slaved to the Leeds and Northrup Recorder. The Polar Recorder in conjunction with the Rotating head provide an integrated rotating motion to the transducer. The receiving system directly records the amplitude versus angular displacement on the Polar Recorder. The beam patterns obtained experimentally are shown in Appendix I. The beam patterns were used to obtain the directivity index for use in Euqation (6-1). The directivity indexes are shown plotted in Fig. 6-20. The directivity index is given as:

$$DI = 10 \log D_r \quad (6\text{-}2)$$

where D_r = Directivity Ratio

The directivity ratio is obtained from the beam patterns. On log-log paper, the db. down from the peak response is plotted versus angular position of the transducer. The areas under the curves are calculated and the directivity ratio is given as the sum of the areas under the curves to twice the total area.

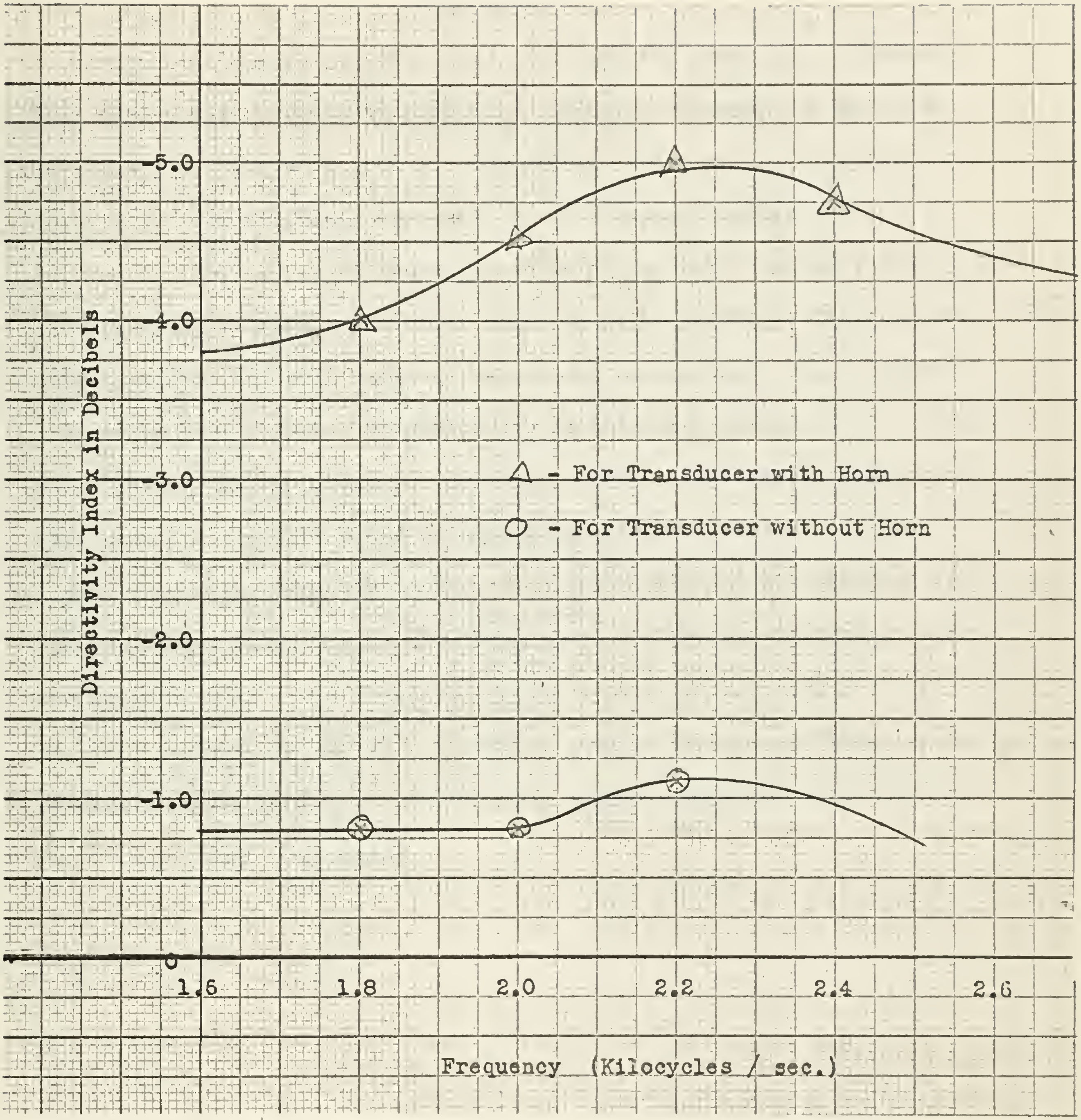


Fig. 6-20. Directivity Index Curves.

CHAPTER VII

CONCLUSIONS

Fig. 6-11 showed that the horn does make the transducer a broader band device but the gain in resistive loading was disappointingly low. This was also brought out in the theoretical shortcomings discussed in Chapter V. The frequency response Fig. 6-19 showed higher gain but a portion of this is due to the directivity of the horn. Based on these findings and the fact that longer horns would be needed to increase the effectiveness, the horn does not seem to be particularly well adapted to underwater sound needs. Shipboard use is ruled out because of the large sizes that would be needed for effective horns. There is a possibility that some use could be found in coastal monitoring systems. It is believed that more effective horns can be built based on the study of the theory. Horns built longer with less flare and greater thickness could add greater gain. Some research should be done in this area. It is doubtful whether the use of a ponderous horn would have any economical advantage in comparison to building the larger transducer that would be needed for lower frequencies.

APPENDIX I

BIBLIOGRAPHY

1. Olson, H. F. and Massa, F., Applied Acoustics, P. Blakiston's Son, 1939.

2. Swenson, G. W., Principles of Modern Acoustics, D. Van Nostrand, 1953.

3. Kinsler, L. E. and Frey, A. R., Fundamentals of Acoustics, John Wiley & Sons, 1950.

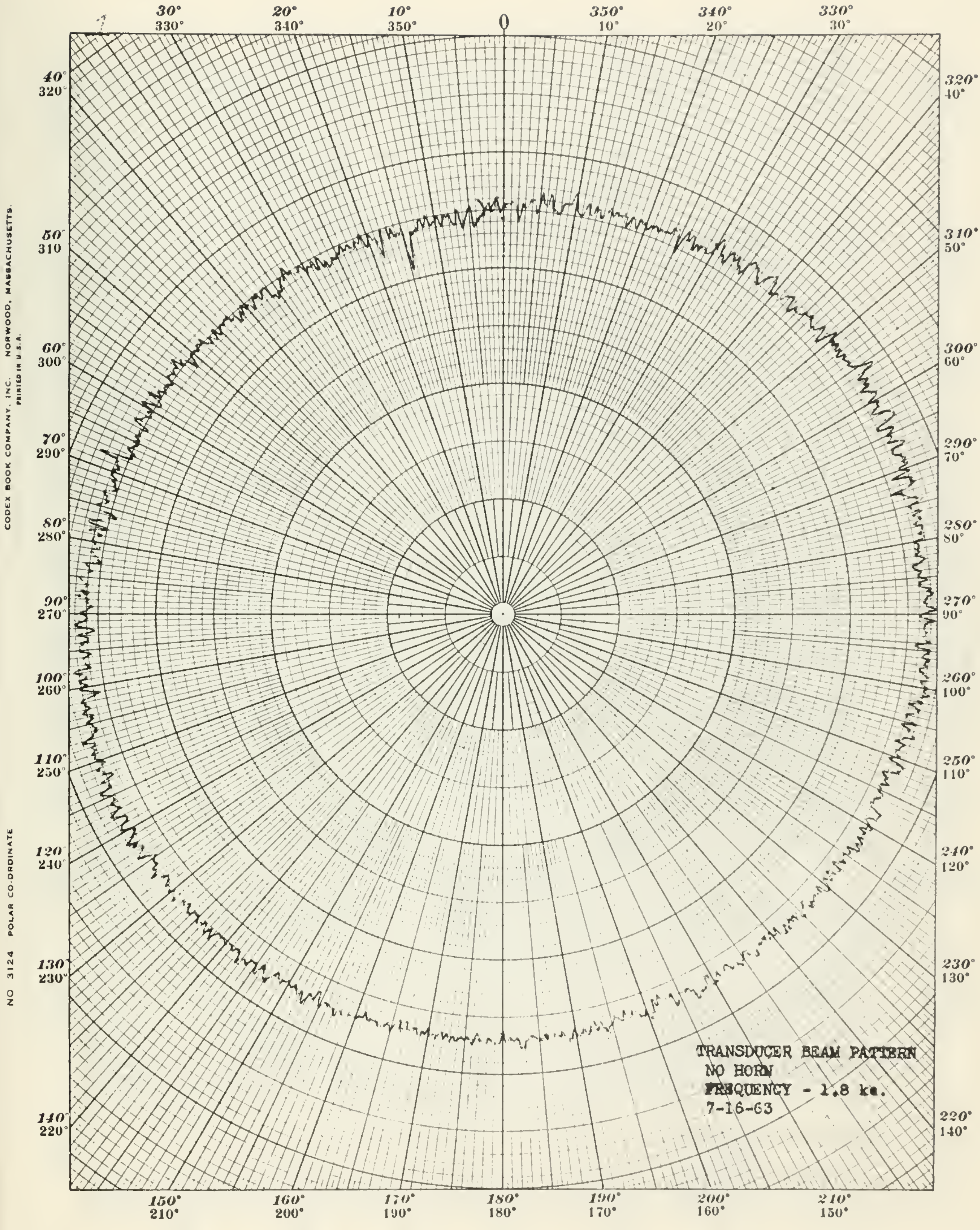
4. Horton, W. J., Fundamentals of Sonar, United States Naval Institute, 1957.

SELECTED READING

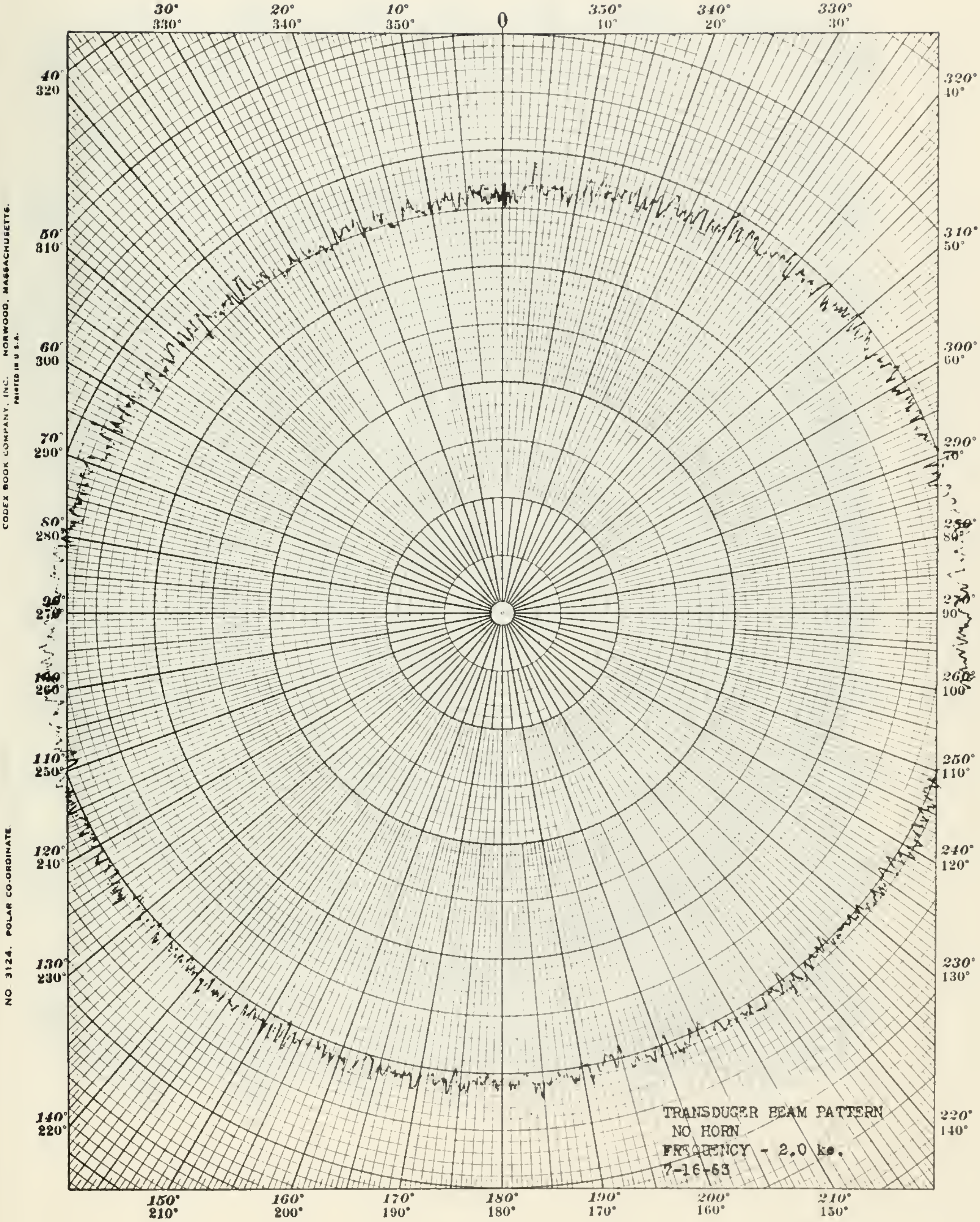
1. Hanna, C. R., Theory of the Horn-Type Load Speaker, Journal of the Acoustical Society, October (1930) p 150-156.

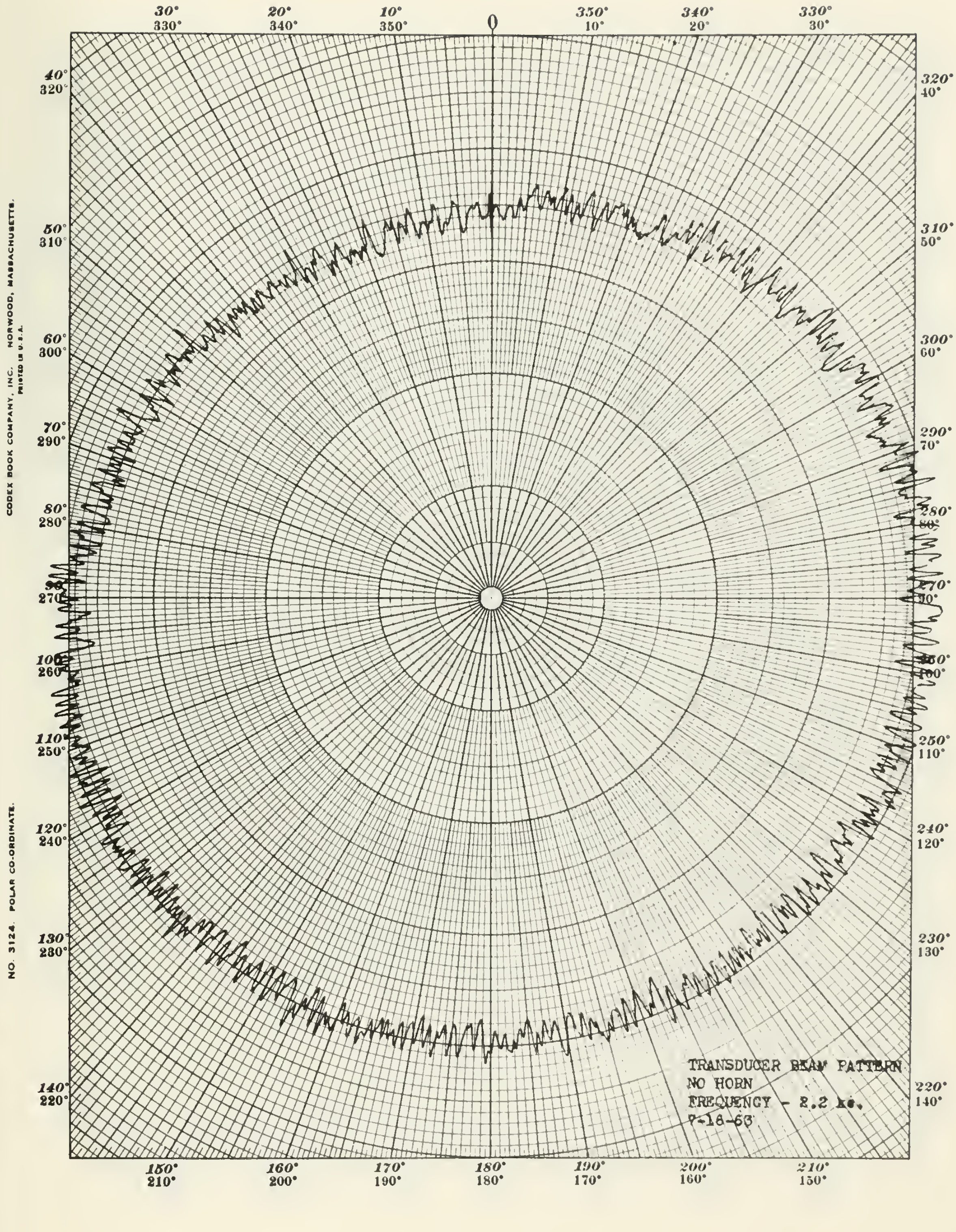
2. Hall, W. M., Comments on the Theory of Horns, Journal of the Acoustical Society, April (1932) p 553-561.

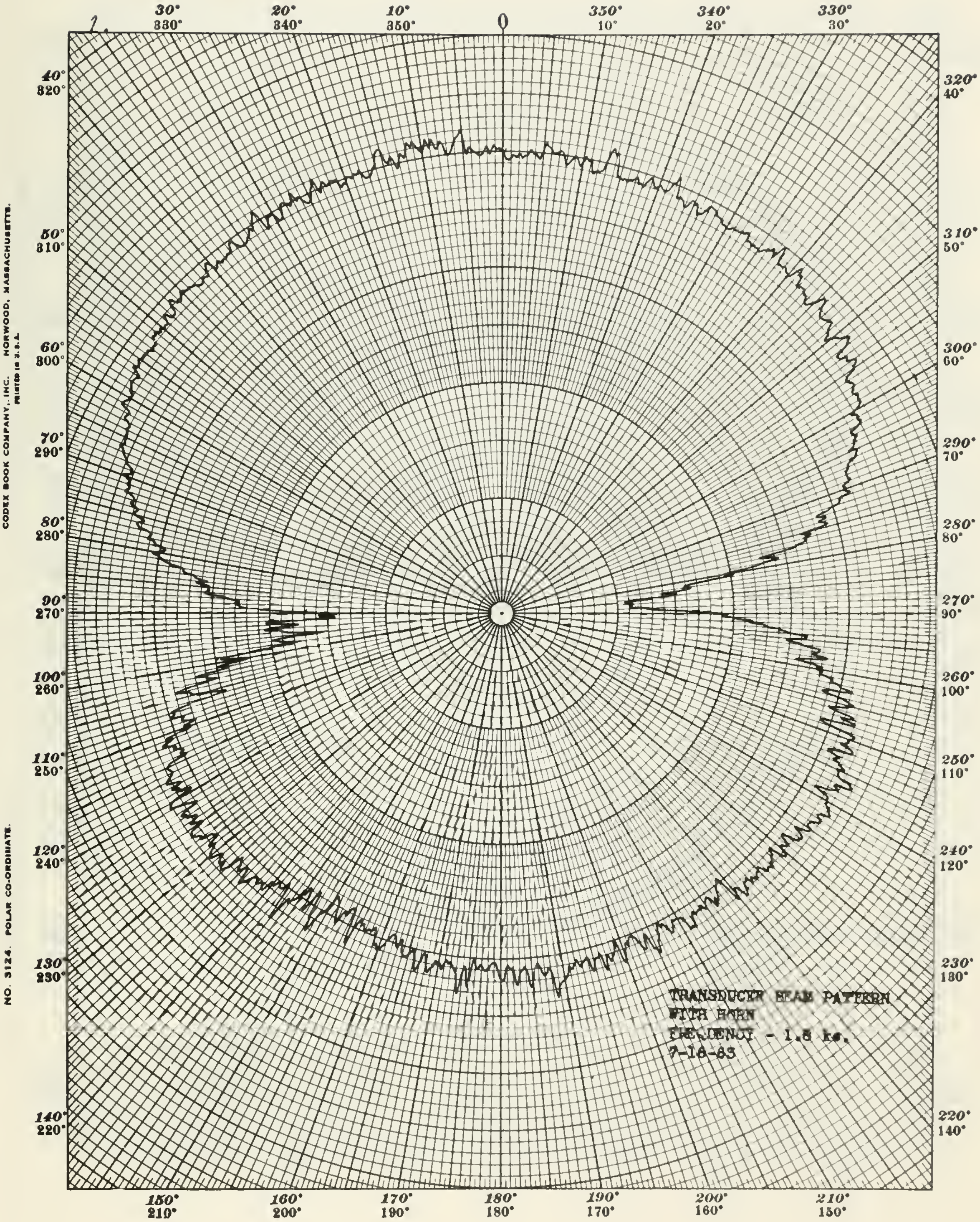
3. Klipsch, P. W., A Note on Acoustic Horns, Proceedings of the I.R.E., July (1945), p 447-449.

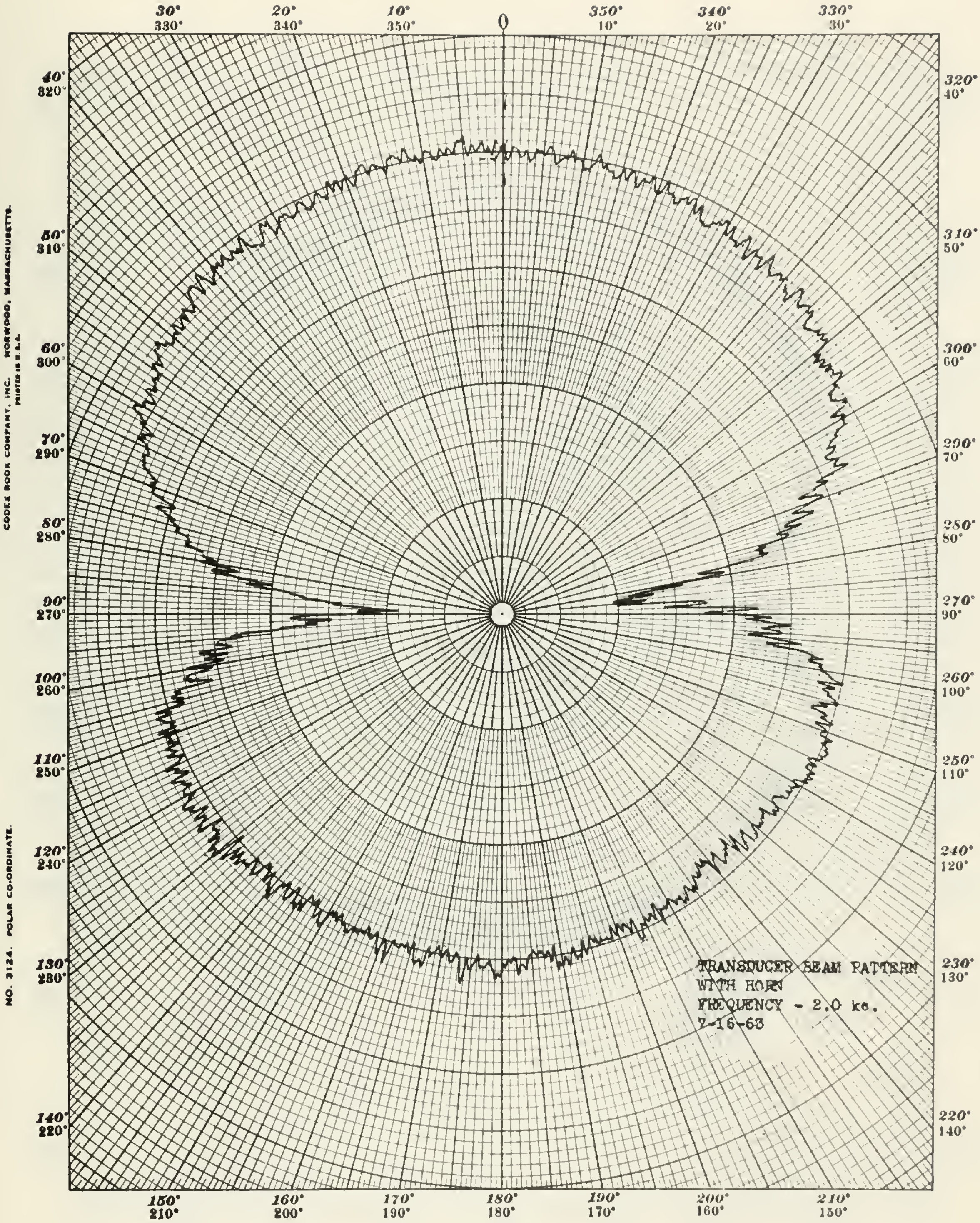


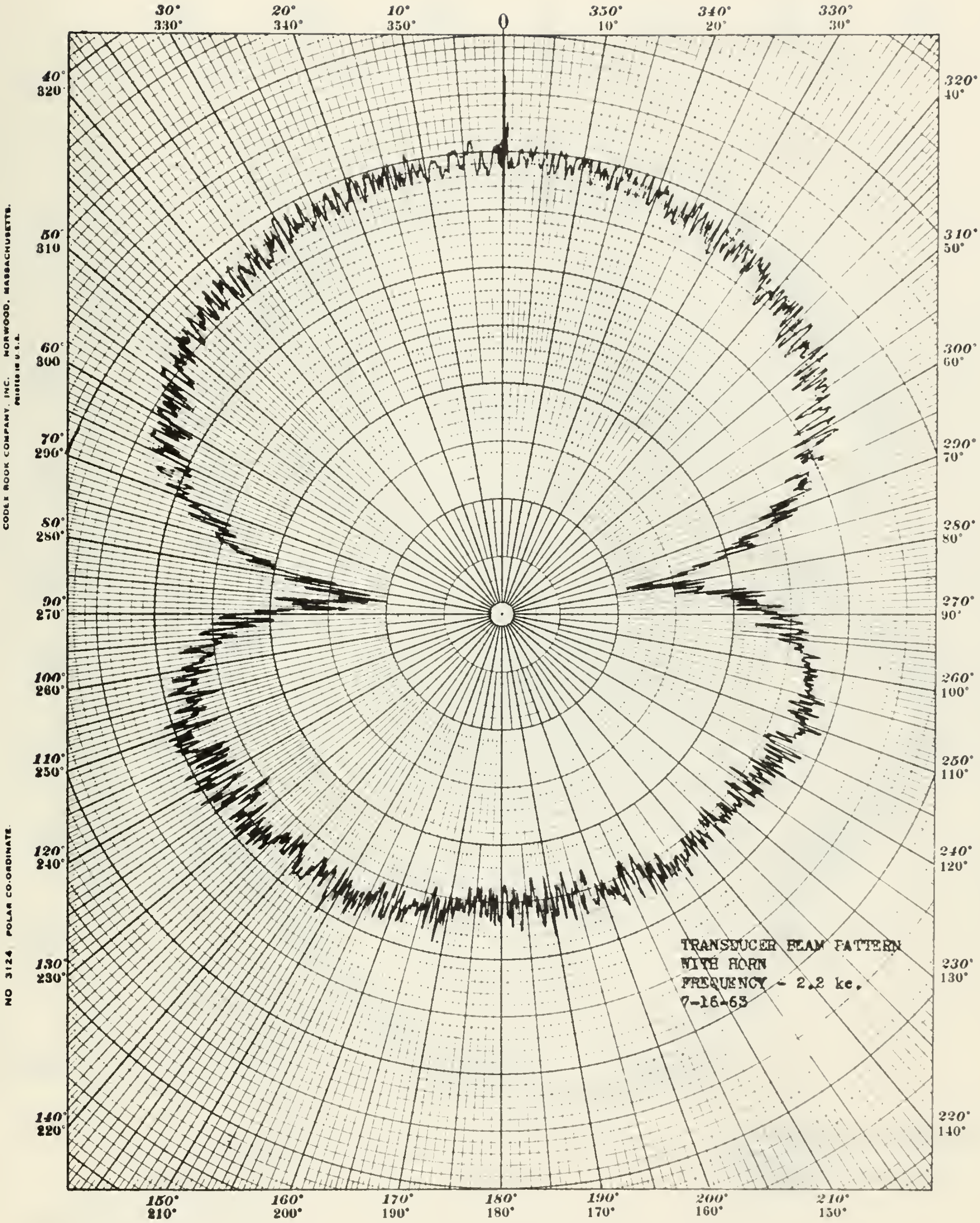
TRANSDUCER BEAM PATTERN
NO HORN
FREQUENCY - 1.8 kc.
7-16-63


TRANSDUCER BEAM PATTERN
NO HORN
FREQUENCY - 2.0 kc.
7-16-63
NO 3124. POLAR CO-ORDINATE.
CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS.
PRINTED IN U.S.A.

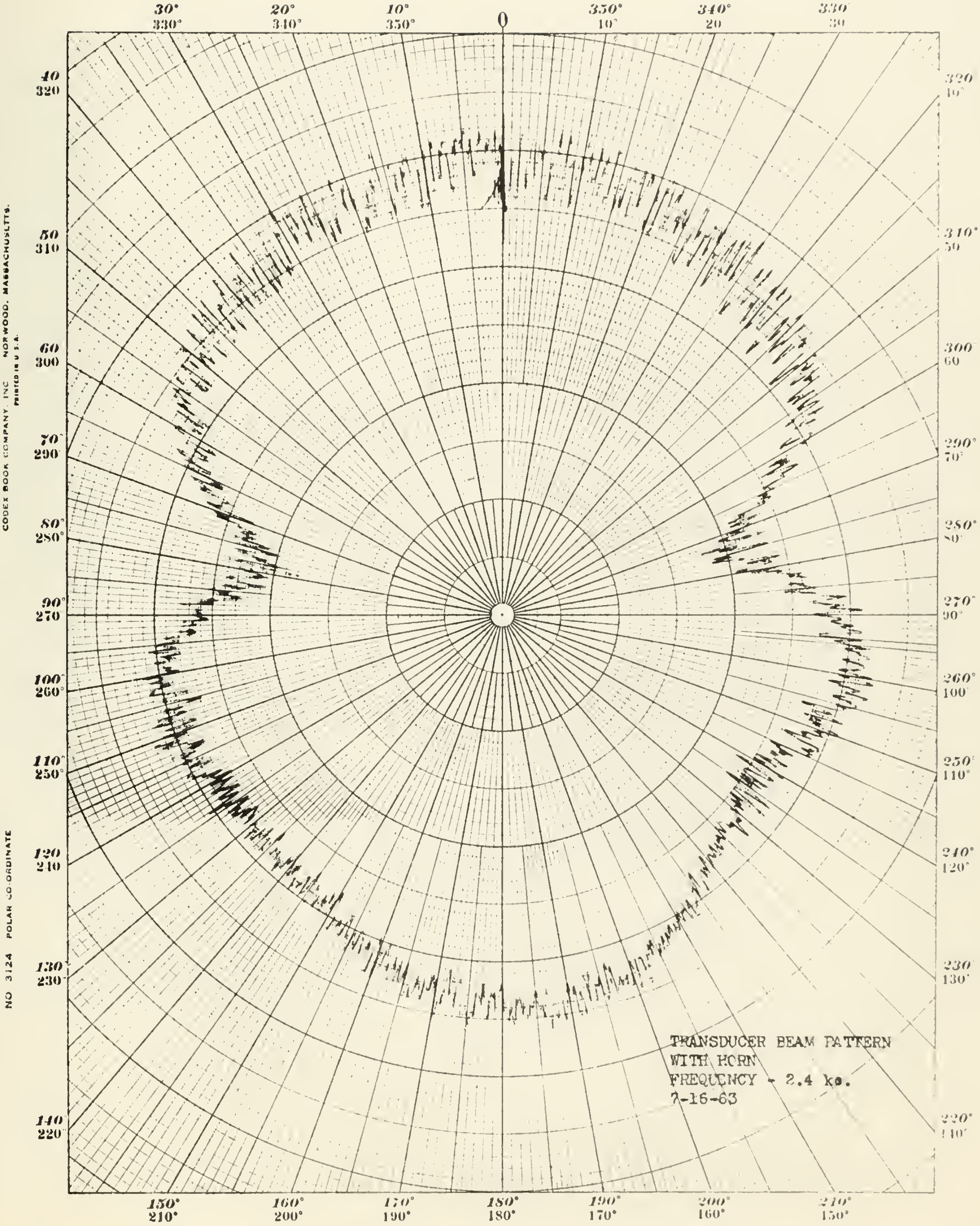

TRANSDUCER BEAM PATTERN
NO HORN
FREQUENCY - 2.2 kc.
7-18-63
NO. 3124. POLAR CO-ORDINATE. CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS.
PRINTED IN U.S.A.


TRANSDUCER BEAM PATTERN
WITH HORN
FREQUENCY - 1.8 kc.
7-16-63
NO. 3124. POLAR CO-ORDINATE. CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS.
PRINTED IN U.S.A.


TRANSDUCER BEAM PATTERN
WITH HORN
FREQUENCY - 2.0 kc.
7-16-63
NO. 3124. POLAR CO-ORDINATE.
CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS.
PRINTED IN U.S.A.



TRANSDUCER BEAM PATTERN
WITH HORN
FREQUENCY - 2.2 kc.
7-16-63
NO 3124 POLAR CO-ORDINATE. CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS. Printed in U.S.A.


TRANSDUCER BEAM PATTERN
WITH HORN
FREQUENCY - 2.4 kc.
7-16-63
CODEX BOOK COMPANY, INC. NORWOOD, MASSACHUSETTS.
NO 3124 POLAR CO-ORDINATE